DIRECT AND ALTERNATING CURRENT POTENTIOMETER MEASUREMENTS

A SERIES OF MONOGRAPHS
ON
ELECTRICAL ENGINEERING

Under the Editorship of
H. P. Young, M.I.E.E., M.A.I.E.E.
Head of the Electrical Power and Machinery Section,
The Polytechnic, London

VOLUME I

THEORY OF ALTERNATING CURRENT WAVE-FORMS
By Philip Kemp, M.Sc.Tech., M.I.E.E., A.I.Mech.E., M.A.I.E.E.

VOLUME II

CATHODE RAY OSCILLOGRAPHY
By Professor J. T. MacGregor-Morris, M.I.E.E., and J. A. Henley, M.Sc. (Eng.)

VOLUME III

DIELECTRIC PHENOMENA IN HIGH VOLTAGE CABLES
By D. M. Robinson, Ph.D., M.S., B.Sc., A.M.I.E.E., Assoc.A.I.E.E.
With a Foreword by P. V. Hunter, C.B.E., M.I.E.E.

VOLUME IV

DIRECT AND ALTERNATING CURRENT POTENTIOMETER MEASUREMENTS
By D. C. Gall, F.Inst.P.
With a Foreword by Professor S. Parker Smith, D.Sc., M.I.E.E., [illegible]

In Preparation

ALTERNATING CURRENT WAVE-FORMS IN PRACTICE
By Philip Kemp, M.Sc.Tech., M.I.E.E., A.I.Mech.E., M.A.I.E.E.

RADIO FREQUENCY BRIDGE AND RESONANCE METHODS
By L. Hartshorn, D.Sc.

FREE ELECTRON ENGINEERING
By C. R. Dunham, B.A., and E. C. S. Megaw, B.Sc., D.I.C., A.M.I.E.E.
With a Foreword by Clifford C. Paterson, O.B.E., D.Sc., M.Inst.C.E., M.I.E.E.

ELECTRIC POWER SYSTEMS
By H. P. Young, M.I.E.E., M.A.I.E.E.

TELEVISION ENGINEERING
By D. C. Espley, M.Eng., A.M.I.E.E.
With a Foreword by Clifford C. Paterson, O.B.E., D.Sc., M.Inst.C.E., M.I.E.E.

HIGH VOLTAGE TRANSIENTS IN POWER SYSTEMS
By H. Morgan Lacey, B.Sc. (Eng.), A.M.I.E.E.

RADIO FREQUENCY OSCILLATORS
By H. A. Thomas, D.Sc., A.M.I.E.E.

MAGNETIC MATERIALS AND TESTING
By C. E. Webb, B.Sc. (Eng.), A.M.I.E.E.
With a Foreword by Professor G. W. O. Howe, D.Sc., M.I.E.E.

Other volumes are in course of preparation, and detailed particulars will be sent free from time to time to anyone sending a request.

A SERIES OF

MONOGRAPHS ON ELECTRICAL ENGINEERING

Under the Editorship of

H. P. YOUNG, M.I.E.E., M.A.I.E.E.

VOLUME IV

DIRECT AND ALTERNATING CURRENT POTENTIOMETER MEASUREMENTS

BY

D. C. GALL, F.INST.P.

WITH A FOREWORD BY

S. PARKER SMITH

D.SC., M.I.E.E., ASSOC.M.INST.C.E.

Professor of Electrical Engineering, Royal Technical College, Glasgow

LONDON

CHAPMAN & HALL I

11 HENRIETTA STREET

1938

First published 1938

Printed in Great Britain by Butler & Tanner Ltd., Frome and London
Bound by G. & J. Kitcat Ltd., London. Flexiback Binding Patent No. 441294

EDITORIAL PREFACE.

The advances made within the realm of electrical engineering during the twentieth century have been phenomenal.

In the field of electric power supply the National Electricity Scheme has been conceived, designed and put into commission, thus increasing the reliability and availability of the service. Meanwhile the allied sciences of electrical communications and high frequency technique have moved forward with gigantic steps; indeed, it is hardly too much to say that, due to the advent of the electronic tube and its application in both the power and communications branches, certain aspects of electrical engineering practice have been completely revolutionized.

In view of these epoch-making advances, it is not surprising that the electrical engineer whose college days are some years behind him finds it necessary to do a considerable amount of reading in order to keep abreast of modern developments, while the student who is reading for a University degree is nowadays expected to possess at least an outline of the field of electrical physics in addition to a comprehensive knowledge of electrical technology.

While, however, the literature dealing with these advances is both comprehensive and voluminous, it is also scattered and thus available only by extensive literary research. The aim of each monograph published in this series is to give a modern orientation of a particular subject within the confines of a small book, thus obviating the necessity for searching through the transactions of innumerable learned societies. At the same time, for those wishing further to extend their knowledge, each monograph contains references to the more important publications relating thereto.

The satisfactory accomplishment of this object necessarily implies that the author must fulfil the dual function of collator and interpreter; for this reason, one of the editor's most important tasks has been to induce acknowledged authorities to write for this series. In this respect, he has been fortunate beyond his expectation, and each monograph has been written by an author who is eminent in his chosen field and thus writes with the authority which is the result of intimate knowledge.

It is hoped that the Monographs will succeed in filling a gap which has hitherto existed in scientific literature.

H. P. YOUNG.

FOREWORD.

In connexion with the matter of electrical measurements Great Britain has always held a fortunate position. Not onl had she her share of pioneers when electricity became an exac science, but many of her eminent scientists have been closel associated with the subsequent development of electrica measuring instruments. The work of such scientists has bee unique, where it has extended from the establishment of th underlying principle to the design, construction and calibratio of the new instruments. Such versatility rather recalls th early days of printing, when the same scholar would write o translate, compose, print and publish.

Among the well-known instrument designers and maker of the present day the author of this book, Mr. D. C. Gal has certainly earned a place. Those who have the privilege o knowing him will readily agree that the subject of precisio measurements is much more than a commercial affair wit him. His keenness to develop methods or instruments t satisfy given requirements and to assist the user to overcom difficulties will also be evident to the reader. One illustratio alone need be cited—the alternating current potentiomete As an instrument of precision, many have given it up owin to difficulties associated with its use. The author recogniz these difficulties and faces them. He designs an instrume with an accuracy within the range of reasonable attainme but plainly sets forth the conditions to be fulfilled. The read thus appreciates that a.c. measurements cannot be carrie out with the same degree of accuracy as d.c. measurements and that in order to obtain what is reasonably possible wit alternating current, requirements such as steady condition

patience, skill and care must be met. These facts the writer knows from experience and the success gained thereby has proved to him that the a.c. potentiometer is too useful an instrument to be discarded.

We hope that the author has succeeded in imparting some of his own outlook on precision measurements to his readers, and so developed in them a critical faculty for which they will always be grateful.

S. PARKER SMITH.

ROYAL TECHNICAL COLLEGE,
GLASGOW.

AUTHOR'S PREFACE.

THIS book is due to the encouragement of Mr. H. P. Young and has for its purpose the recording of the experience which has come my way in a close association with the art of electrical measurement. Now that it is concluded, my grateful thanks are due to him for his persistence, without which I had wearied of the discipline of putting down in writing the principles and practice of this branch of the art as I have understood them.

I am grateful to Dr. L. G. A. Sims for reading the draft proof and for many needful corrections. To Mr. D. Rutenberg I am greatly indebted for his long search into the historical archives of the subject and for his systematic help in many other directions. I have to thank Mr. D. Connelly for a great deal of help and the fine examples of actual measurement carried out at the Royal Technical College, Glasgow, upon which I have drawn freely, with the permission of Professor Parker Smith from whom I have received help and encouragement at every stage.

Since the subject-matter deals with methods of measurement and the apparatus involved therein, there is little mathematical work. A knowledge of alternating current theory is requisite in applying the results of measurements made with the a.c potentiometer. Chapter XIII is included to give a summary of the most usual calculations which are involved and to assist students in applying the rules of vector algebra, which they are assumed to have already learnt, to the interpretation of the results of measurements made with any type of a.c. potentiometer. It is also intended to explain why no distinction is made in the symbols used throughout between vector and simple numerical values of voltage and current. The natur

of any quantity will be at once obvious from the measurement and the use of dots to indicate vector values seems to have no practical value and has already been discontinued in recent works on circuit theory. Students who receive their early instruction in alternating current circuit theory by the use of the simple a.c. potentiometer miss the confusion which a mass of blackboard symbols so often invoke, and in the author's experience a foundation of clearer understanding can be laid by a practical measurement of the vector values, than any voltmeter and ammeter experiment can possibly give. The usual pitfalls of applying vector algebra to alternating current calculations do not arise because the nature of the qualities involved are understood when they can be measured in vector form and are not just blackboard symbols derived by faith from a theorem taken on trust.

There is some repetition in the text. The apology for this is that it is appropriate to different sections into which the chapters have been divided and is intended to make the reading of each more complete without back reference.

D. C. GALL.

Nov., 1937.

CONTENTS.

CHAPTER IX.

CHAPTER X.

CHAPTER XI.

CHAPTER I.

PRELIMINARY.

The fundamental units of electrical measurements, the ohm, the volt and the ampere, are determined initially by state standardizing bureaux established in different parts of the world. The physical conditions under which these determinations are made are fixed by international convention; and close agreement is maintained between the several units by the various countries. To ensure uniformity of magnitude of the units employed in science and industry it is necessary to correlate the indications of instruments and apparatus used for measurement purposes with the magnitudes of the units obtained by the absolute determinations of the standardizing bureaux. From the practical point of view the most convenient units to use for this correlation are the unit of resistance, the ohm, and the unit of potential difference, the volt, for it is possible to construct resistors and standard cells, relatively robust mechanically, which retain their electrical properties without variation of value over considerable periods of time. The apparatus used to act as intermediary between the practical instruments and the absolute standards consists of a potentiometer and a standard cell.

The potentiometer, as its name implies, is an instrument which is used for the measurement of potential differences. In addition to providing the means of correlating the practical electrical standards with the absolute determinations of the electrical units, its range of measurement may be extended to make it suitable for the precision measurement of current, power, and the other electrical units. The modifications of the potentiometer principle which have been developed to employ the high precision of which this method is capable, together with the practical considerations arising out of the

CHAPTER I.

PRELIMINARY.

The fundamental units of electrical measurements, the ohm, the volt and the ampere, are determined initially by state standardizing bureaux established in different parts of the world. The physical conditions under which these determinations are made are fixed by international convention; and close agreement is maintained between the several units by the various countries. To ensure uniformity of magnitude of the units employed in science and industry it is necessary to correlate the indications of instruments and apparatus used for measurement purposes with the magnitudes of the units obtained by the absolute determinations of the standardizing bureaux. From the practical point of view the most convenient units to use for this correlation are the unit of resistance, the ohm, and the unit of potential difference, the volt, for it is possible to construct resistors and standard cells, relatively robust mechanically, which retain their electrical properties without variation of value over considerable periods of time. The apparatus used to act as intermediary between the practical instruments and the absolute standards consists of a potentiometer and a standard cell.

The potentiometer, as its name implies, is an instrument which is used for the measurement of potential differences. In addition to providing the means of correlating the practical electrical standards with the absolute determinations of the electrical units, its range of measurement may be extended to make it suitable for the precision measurement of current, power, and the other electrical units. The modifications of the potentiometer principle which have been developed to employ the high precision of which this method is capable, together with the practical considerations arising out of the

many forms of measurements which can be made of uni-directional and alternating electrical quantities, constitute the subject-matter of the ensuing chapters.

The potentiometer consists essentially of a resistance through which a constant current may be made to flow. The resulting potential drop in this resistance is first standardized by comparison with the electro-motive force of the standard cell and, secondly, is compared against the unknown potential difference which has to be determined. In this manner the unknown potential is in effect compared with the electro-motive force of the standard cell, and is thus determined indirectly against the absolute standard of potential difference. As an adjunct necessary and common to all potentiometers, the standard cell will be considered first.

The Standard Cell.—The standard cell is a simple assembly of chemicals which can be prepared under suitable conditions to reproduce the e.m.f. at its terminals with great consistency. Two principal types of cell have been devised, the first by Latimer Clark,[1] the second by Edward Weston.[2] Of these, the latter has almost entirely superseded the former, due to the larger temperature coefficient and inferior degree of reproducibility of e.m.f. of the Clark cell. Attention will therefore be confined to the Weston type which, because of its constituents, is known also as the cadmium cell. Standard specifications for the manufacture of cadmium cells have been drawn up by the National Physical Laboratory, the Bureau of Standards, Washington, and other standardizing bodies. A full description and extensive bibliography will be found in the "Dictionary of Applied Physics" (Glazebrook), vol. 2, pp. 260–73. The components of a cadmium cell are illustrated in Fig. 1.

Originally these cells were made with a neutral electrolyte, and the e.m.f. was then 1·01830 volts at 20° Centigrade. In the above reference it will be seen that the permanence of the cells is much improved by making the electrolyte acid although this has the effect of lowering the e.m.f. These acid cells have an e.m.f. of 1·01824 volts at 20° Centigrade, falling by 40 microvolts per degree Centigrade rise of temperature. The positive limb of the cell contains mercury and mercurous sulphate; the negative limb an amalgam of cadmium and

mercury ; the whole is filled with an electrolyte of acid cadmium sulphate solution. The permanence of these cells is of a very high order and provides a fundamental standard for practical electrical measurements when used in conjunction with a precision potentiometer and standard resistance. In practice difficulty is encountered in obtaining chemicals of sufficient purity. Commercially manufactured cells should however not vary by more than a few parts in 100,000 from the nominal

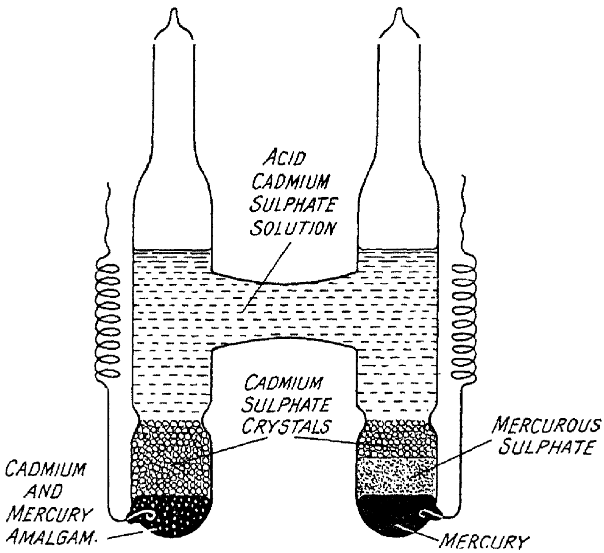


FIG. 1.—Weston cadmium standard cell.

value. The life of such cells is a little uncertain and their value should be checked at least once every two years.

The function of the standard cell is to provide a source of e.m.f. ; it must not be used as a source of current. Since the standardization of potentiometers depends upon a null balance the current taken from the cell is negligible. Care must always be taken so that only the smallest possible current is drawn from the cell while balance is being obtained. One microampere will probably lower the voltage by a millivolt due to the internal resistance. Should the cell be accidentally short circuited it will recover its correct voltage after a rest. The internal resistance of the cells is of the order of 1,000 ohms ;

this tending to increase with time. It frequently causes the cell to become unusable after a period of years, for when the resistance is too high there is a loss of sensitivity in balancing the cell on the potentiometer.

It is useless to try to test a standard cell by means of an ordinary voltmeter, for its internal resistance will cause a large drop in the terminal potential difference below the e.m.f. In addition, the current taken will be far too large and consequently cause damage to the cell.

When using a standard cell it is important that the whole should be at a uniform temperature. The temperature coefficients of the two limbs of the cell are individually quite large but are of opposite sign; their resultant is almost negligible for most purposes. If one limb is at a different temperature from the other, due possibly to the absorption of radiant heat by one side of a cell case unsuitably placed on the test bench, serious disturbance of the e.m.f. of the cell may occur. In work of the highest precision it is usual to immerse the cell in oil which ensures that the whole is at a uniform temperature. For ordinary use it is a good plan to place the cells in a box with cotton-wool packing.

For convenience in handling, standard cells are mounted in brass or wooden cases, the former where it is desirable that they should be oil immersed. Provision should be made for the insertion of a thermometer to measure the temperature of the cell when it is desired to apply the temperature correction. Each case may contain either one or two cells. The advantage of a double cell is that one of them can always be used to check the other. If the two are in exact agreement it is fairly certain that they are in good condition.

Standard cells from their inherent nature require careful handling. Although the chemicals are very firmly crystallized into place, very violent treatment may displace them and ruin the cell by allowing opposite sides to mix.

Other types of standard cells which have been used are given in the following table. The Weston unsaturated cell is in wide use in the United States of America. It has the advantage of a very small temperature coefficient, but is not considered so permanent as the saturated acid cell.

Name of Cell.	Positive Electrode.	Negative Electrode.	Electrolyte.	International Voltage 20° C.	Temperature coefficient MV per ° C.
Daniell, 1836	Copper	Zinc	Zinc sulphate	1·08	+ 34
Clark, 1872	Mercury	Zinc amalgam	Saturated zinc sulphate	1·4271	− 1100
Weston saturated (neutral), 1892	Mercury	Cadmium amalgam	Cadmium sulphate saturated	1·01830	− 40
Weston unsaturated, 1903	Mercury	do.	Cadmium sulphate saturated at 4° C.	1·0186	− 10
Weston Acid, 1908	Mercury	do.	Cadmium sulphate saturated 1/10 N sulphuric acid	1·01824	− 40

The above values are in international volts. In 1940 it is proposed to adopt absolute values for the units. This means that the standard cell will have a value of 1·01860 absolute volts at 20° C. The absolute volt is 370 microvolts lower than the international volt.

Similarly the absolute ohm has a different value from the international ohm in which all resistances are now adjusted. A 1-ohm resistance in the present international units becomes 1·00048 ohms in absolute units.

If, therefore, measurements are made with the new absolute values considerable care will be necessary in applying the proper corrections.

One absolute ampere through an international ohm will give a voltage 480 microvolts high of the absolute volt. Measured upon a potentiometer set up in absolute volts this would give an error of 0·048 per cent. Measured on a potentiometer set up in international volts this would give an error of 0·012 per cent. in the absolute value of the current.

NOTES AND BIBLIOGRAPHY.

[1] Clark cell: due to Latimer Clark, 1872; for description see "On a Voltaic Standard of Electromotive Force," *Proc. Roy. Soc.*, 1872, xx, 448; also "On a Standard Voltaic Battery," *Philos. Trans.*, 1874, clxiv, 1–14.

[2] Weston cadmium cell: due to Dr. Edward Weston, 1893; U.S. Patent 494827. For early description see "The Weston Standard Cell," *Elect. Engr. New York*, 1893, xv, 355–6; also *Electrician*, 1893, xxx, 71; also *Bur. of Stand.*, iv, 1.

CHAPTER II.

THE D.C. POTENTIOMETER.

In its simplest form the d.c. potentiometer consists of a uniform slide wire as illustrated diagrammatically in Fig. 2. The slide wire is connected to a battery through a variable resistor, and the voltage drop in the wire due to the current flowing is used for the comparison of unknown voltages with the known e.m.f. of a standard cell.

If the total resistance of the circuit is R ohms and the battery e.m.f. is E volts, the current flowing will be $I = E/R$ amperes, and the voltage drop on any length of the slide wire will be $Ir = Er/R$ volts, where r represents the resistance

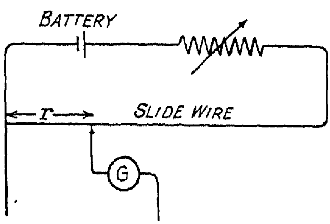


FIG. 2.—Simple slide-wire potentiometer.

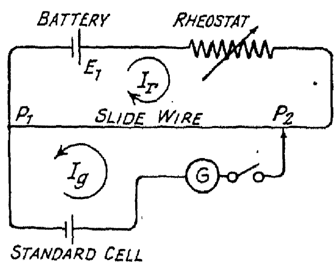


FIG. 3.—Simple slide-wire potentiometer with standard cell.

of this length of the slide wire. If the wire has a uniform cross-section, and hence the same resistance per unit length, the voltage drop will be proportional to the length of wire between the potential points. Thus a voltage drop can be selected between one end of the wire and the slider, which latter can be moved from end to end of the wire. If now the circuit is completed as shown in Fig. 3 by connecting the standard cell through a galvanometer and key to the slider and to one end of the slide wire, a point can be found where the voltage drop selected will be equal to the e.m.f. of the standard cell.

This condition will be indicated when the galvanometer shows that no current is passing. The potentiometer is then said to be balanced or standardized against the standard cell.

In practice it is an advantage to make the balance occur at a convenient point on the slide wire by varying the current in the potentiometer circuit. A variable resistor is included in the circuit for this purpose. For example, consider a slide wire of resistance 10 ohms which is provided with a scale 1·5 metres long graduated in millimetres, the wire being connected through a variable resistor Rs to a 2-volt accumulator. The scale would be direct reading in volts if the standard cell were balanced at 1,018 millimetres. This implies that the volt drop on 1,018 millimetres will be $1{\cdot}018 = Er/R$ volts. Since the resistance of the wire is proportional to its length the voltage drop on 1,500 millimetres will be 1·500 volts. The necessary value of Rs will then be such that $\frac{1{\cdot}5}{10} = \frac{2}{10 + Rs}$ whence $Rs = 3{\cdot}3\dot{3}$ ohms. The limiting values to which Rs should be adjustable are determined by the variations which are likely to occur in the battery voltage. These variations might range from 2·2 to 1·8 volts if an accumulator is used, so that in order to permit the slide-wire potentiometer to be used for direct reading with 1 millimetre on the scale equivalent to 1 millivolt, the resistance Rs should be variable between the values

$$Rs \text{ max.} = \frac{22}{1{\cdot}5} - 10 = 4{\cdot}67 \text{ ohms}$$

and

$$Rs \text{ min.} = \frac{18}{1{\cdot}5} - 10 = 2{\cdot}0 \text{ ohms.}$$

Thus a fixed resistor of 2 ohms and a 2·8 ohms variable section might be used. The rheostat of any potentiometer should be capable of providing about 20 per cent. variation in the total resistance of the potentiometer to compensate for changes in the accumulator voltage.

When the simple slide-wire potentiometer is standardized in this way, any other voltage within its range may be measured by substituting the unknown voltage for the standard cell and balancing by movement of the slider until the galvanometer shows no current to be flowing.

From the practical point of view the use of a slide wire with very low resistance is objectionable, for it requires a large current to produce the necessary volt drop, which results in unsteadiness, and also discharges the battery quickly. On the other hand, a high-resistance wire is much more liable to mechanical damage unless the potentiometer is suitably constructed, and is usually of inferior uniformity to the low-resistance wire. Particular care should be taken that the slider cannot be pressed into the wire by the user.

Theory of Potential Balance.—In the simple potentiometer circuit shown in Fig. 3, the current flowing through the galvanometer can be calculated by setting up the circuit equations for the currents and voltages in the two meshes formed by the potentiometer circuit and the galvanometer circuit.

Let $R \equiv$ the total resistance of the potentiometer circuit consisting of the slide wire, rheostat and battery

$r \equiv$ the resistance between points P_1 and P_2 in Fig. 3

$r_g \equiv$ the resistance of the galvanometer circuit between the potential points P_1 and P_2 including the internal resistance of the standard cell and any other resistance

$E_1 \equiv$ the e.m.f. of the battery supplying current to the potentiometer

$E_2 \equiv$ the voltage which is acting in the external circuit (from the standard cell).

By Kirchoff's laws the two voltages E_1 and E_2 will be made up by the products of the currents and resistances in their respective circuits.

If I is the current in the potentiometer circuit and I_g the current in the galvanometer circuit

$$E_1 = IR + I_g r$$
$$E_2 = I_g(r_g + r) + Ir$$
$$\frac{E_2}{E_1} = \frac{I_g(r_g + r) + Ir}{I.R + I_g r}$$

whence
$$I_{\cdot} = \frac{I\left(\frac{E_2}{E_1}R - r\right)}{r\left(1 - \frac{E_2}{E_1}\right) + r_g}$$

This is the galvanometer current in terms of the current in the potentiometer. Eliminating I gives

$$I_g = \frac{E_1 r - E_2 R}{r^2 - R(r_g + r)}$$

At balance $I_g = O$

$$\therefore \frac{E_2}{E_1} = \frac{r}{R}.$$

The sensitivity with which a balance can be obtained will depend upon the galvanometer current which flows when the circuit is slightly out of balance.

It is seen from the above that the circuit will be balanced when

$$\frac{E_2}{r} = \frac{E_1}{R}.$$

The current I in the resistance r will then be equal to $\frac{E_1}{R}$ because no current is entering or leaving through the galvanometer circuit. The galvanometer current which flows when a small change is made in the balance condition of any potentiometer is best calculated by use of the Compensation Theorem. This theorem is as follows:

"If a network is modified by making a change Δr in the resistance of one of its branches the current increment thereby produced at any point in the network is equal to the current that would be produced at that point by a compensating e.m.f. acting in series with the modified branch, whose value is $-I\Delta r$ where I is the original current flowing in the modified branch."

This simply means that a current will flow round the galvanometer circuit equal to the out-of-balance voltage divided by the resistance of the galvanometer circuit plus the resistance between the potential points.

That is $I_g = \frac{\Delta E}{r_g + r}$ where $\Delta E = -I\Delta r$.

The current in the potentiometer circuit will also be disturbed by a change in value equal to $\frac{\Delta E}{R}$.

If the value of r is increased above the balance position,

Δr is positive and current will flow from the potentiometer into the galvanometer circuit and the current in the potentiometer wire will be reduced, but if the value of r is decreased below the balance point Δr is negative and current will flow into the potentiometer circuit from the galvanometer circuit. That is, the deflexion of the galvanometer will be reversed. The current in the slide wire will be increased because ΔE will then be positive.

The sensitivity of the potentiometer to the measurement of small voltages is therefore readily estimated from the galvanometer sensitivity.

If the galvanometer has a sensitivity of 150 millimetres per microampere and a resistance of 100 ohms in a potentiometer circuit where r is 50 ohms and the external resistance is 1,000 ohms, the current through the galvanometer with 1 millivolt out of balance would be

$$\frac{10^{-3}}{100 + 50 + 1000} = 0{\cdot}87 \text{ microamperes.}$$

This would give a galvanometer deflexion of about 130 millimetres. It would therefore be possible to obtain a balance with a precision of rather better than 10 microvolts, giving a sensitivity of 1 millimetre galvanometer deflexion.

It will be obvious from the equation for the galvanometer current that the condition of the circuit to give maximum sensitivity can be calculated, but for general purposes there will be wide variations in the resistance of the external circuit so that the optimum conditions would only apply to special cases where the circuit resistance can be approximately known. The theoretical solution is not the best practical one. This is dealt with in Chapter V.

The Limitations of the Simple Slide Wire.—The standard cell has a value reproducible more accurately than 1 part in 10,000, but the simple slide wire which has formed the basis of the foregoing potentiometer could not be read to less than say 0·5 millimetre or 1 in 2,000 at the 1-volt setting. The non-uniformity of the resistance of the wire would also be such that the resistance per centimetre would probably vary by one or two per cent. so that the true voltage gradient along

the wire would not agree with the millimetre scale. Further, due to unavoidable practical uncertainties in the exact terminating point, the lowest readable voltage would certainly not be less than 1 millivolt, and that would only be observable with a large possible error, so that small voltages could not be read with any degree of precision.

Increasing the Degree of Subdivision.—It becomes necessary, therefore, to extend the degree of subdivision of the potentiometer if any but the crudest measurements are required. The simplest means of extending the subdivisions of potential gradient is to add a set of fixed resistors in series

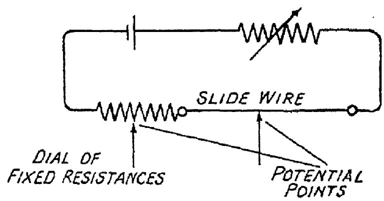


FIG. 4.—Simple potentiometer with two dials.

with the slide wire as shown in Fig. 4. This type of potentiometer was first introduced by Crompton in 1895.

If these resistances are all equal in value to that of the slide wire the degree of subdivision is at once increased in direct proportion to the number of studs, 10 times for 10 studs or 100 times for 100 studs. Further, the precision of subdivision can be greatly improved because each resistance step can be accurately adjusted to equality, and lack of uniformity in the slide wire is of less importance.

The "Resistance" Gradient.—It should be noted that the actual values of the resistances in the potentiometer are of no importance so long as they are equal. This facilitates construction because it is quite easy to adjust a number of resistors to equality with an extremely high degree of precision. The slide wire need not have the same resistance, and it is a definite advantage for the volt drop upon the slide wire to overlap that of the studs, so that if these are arranged to be say 0·1 volt each, the slide wire might conveniently be 0·12 volt. The volt drop on the slide wire can be reduced to any convenient

value by means of a shunt such as is illustrated in Fig. 5, since this only lowers the total resistance and therefore the volt drop between the ends, but not the relative potential gradient along the shunted wire. This is the method usually employed to give the slide wire a convenient volt drop.

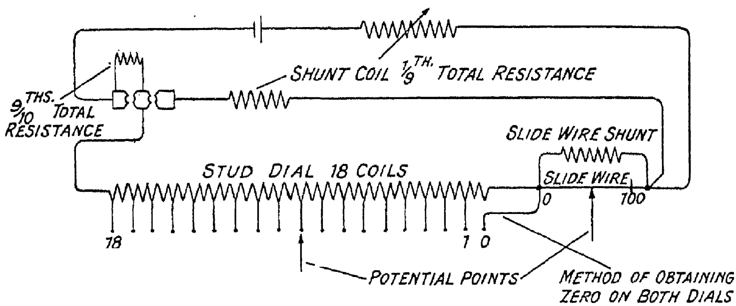


FIG. 5.—Two-dial two-range potentiometer.

Additional Ranges.—The whole potentiometer can also be shunted to alter the voltage range, and it is usual to add a ballast resistance to keep the overall resistance unchanged, when this is done. Fig. 5 shows a simple scheme which allows the potentiometer to be standardized against the standard cell on the unshunted range and to maintain the circuit resistance unchanged when shunted.

By making the shunting resistance 1/9 and the series resistance 9/10 of the resistance of the potentiometer circuit the current will be reduced to 1/10 when the shunt is added. This gives the potentiometer 1/10 of its unshunted range.[1] In order to give a true zero the O stud can be connected as shown in Fig. 5 to the beginning of the slide wire. The first coil is then joined to the slide wire and not to the O stud; in this way the volt drop in the lead between the slide wire and the dial is eliminated. A still better arrangement is that used in the "General Utility" potentiometer in which the slide wire gives not only a zero but also a small negative reading, by means of the tapped shunt on the slide wire.

It should be noted that this small negative reading is obtained by what is, in effect, a cross-over of the two potential points.

The scheme is shown in Fig. 6. This potentiometer has also three ranges obtained by shunting the potentiometer in the manner shown. The feature of the shunting method is that the necessary changes are brought about by one plug only. A further feature is the use of a parallel circuit for the standard cell, so that the volt drop of the potentiometer is standardized without having to set the dials to any particular value, and it can be standardized in this way when used upon any of the three ranges.

The total resistance of the standard cell circuit is really arbitrary and only the ratio of its two parts is important.

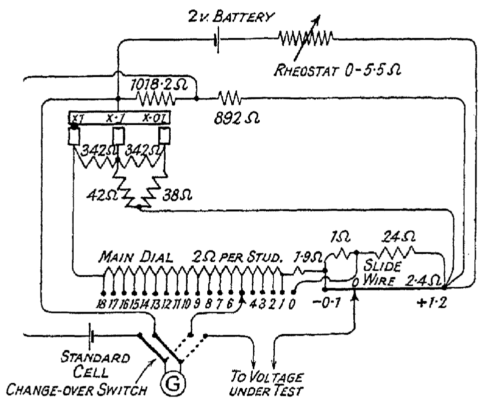


FIG. 6.—General utility potentiometer.

This must equal the ratio of the standard cell voltage to the total nominal voltage across the potentiometer dials, which latter in this case is slightly more than 1·9 volts, due to the slide wire reading more than 0·1 volt.

The single-dial and slide-wire potential gradient, with reducing ratios, provides a potentiometer capable of a potential subdivision of 1 in 10,000 at 1, 0·1 and 0·01 volt. It is usual to have 18 studs on the dial so that a maximum volt drop of 1·9 volts can be obtained, the rheostat making up the difference between that and the 2-volt accumulator voltage. It is not advisable to work the accumulator much below 2 volts for precise work because the current becomes unsteady as the voltage falls. When possible the accumulator, which should

be of ample capacity, should be left connected to the potentiometer for some hours to let the current become steady before using the potentiometer, particularly when work of high precision is contemplated. It is the need for great steadiness which makes the use of too low a resistance circuit undesirable. About 40 ohms is the lowest practical value.

It is always advisable to have some means of reducing the sensitivity of the galvanometer, so that in measuring an unknown voltage the deflexion of the galvanometer can be kept under control.

A switch-controlled series resistance is best for this purpose as it protects the circuit from excessive current better than a galvanometer shunt. A four-position resistance box of 0, 1,000, 10,000, and 100,000 ohms is most suitable. When starting to measure an unknown voltage the full 100,000 ohms should be switched into the galvanometer circuit. The sensitivity of the galvanometer will then be so low that the approximate balance can be easily found. If the polarity of the test voltage is reversed the minimum deflexion of the galvanometer will occur when the potentiometer reads zero. This indicates that the leads to the unknown require transposing. The series resistance must be free from thermo-electric effects particularly in the zero position.

NOTE.

[1] Change of range: This method due to Dr. Rudolf Franke, 1897; see "Ein Kompensator für Spannungs und Strommessungen," *Elektrotech. Z.*, 1897, xviii, 318–20.

THE D.C. POTENTIOMETER (*continued*).

Potentiometers of Higher Precision.—The order of accuracy obtainable with a potentiometer is dependent upon the degree of subdivision of its constituent resistance units. The greater this subdivision, the more definitely can the potential difference between the travelling points be adjusted, and consequently the more accurately can a potential difference be measured. In the simple potentiometers so far described there are two variable potential points and in consequence no more than two dials (one of which may be a slide wire) can be used. The degree of subdivision is therefore limited to the number of steps that it is practicable to construct. To obtain potentiometers capable of measurements of higher precision it is necessary to make certain modifications of circuit and construction which overcome this limitation.

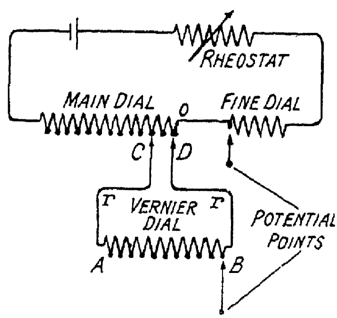


FIG. 7.—The vernier dial.

One of the best methods of obtaining further subdivision of the potential gradient is by means of the Varley vernier principle [1] originally devised for fault localizing on submarine cables.

The scheme, which is shown in Fig. 7, is to shunt two studs of the main dial by a vernier dial, the resistance of which is made the same as that between the two studs which it shunts. This reduces the resistance between the two shunted studs to the same as that of one, and therefore to the same volt drop. This volt drop is subdivided into any number of steps according to the number of coils in the shunting or vernier dial.

The vernier principle can be continued indefinitely and is only limited by the effect of the resistance of the contacts of the vernier dial. These enter into the division of the current in the two paths and in precise instruments play an important part in the accuracy. As the number of dials increases, the resistance of each vernier dial decreases rapidly until its steps become comparable with the contact resistance and therefore lose significance. Fig. 8 shows the circuit of a five-dial instrument of this type with a potential subdivision of 1 in 100,000. Such a potentiometer is often used for fault location on long or low resistance cables.

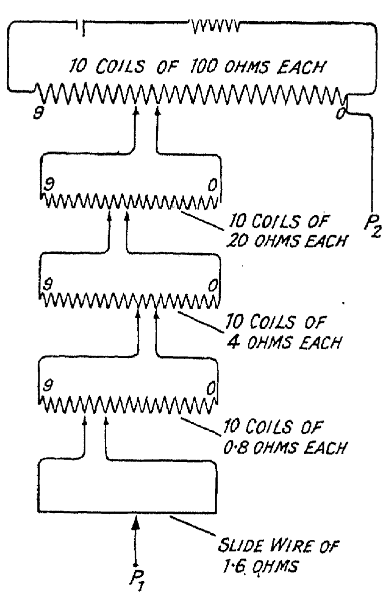


FIG. 8.—Five-dial Varley potential divider.

Fig. 9 shows the schematic arrangement of a potentiometer using a vernier dial and two additional dials. This gives a degree of subdivision of 1 in 180,000 with a very high degree of precision. In addition the special parallel arrangement [2] provides both a true zero and negative potential values on the lowest dial.

The Theory of the Vernier Dial.—In order to achieve a precision of this order, it is necessary to study the circuit more closely. The vernier dial consists of a set of coils formed into a dial. The ends of the resistance formed by the coils are shunted across two coils in the main dial by means of switch contacts which travel along the coils of the main dial. In this way any adjacent pair of the main dial coils are shunted down to a value of resistance equal to one coil. The volt drop across the vernier dial is therefore equal to that of one stud on the main dial, so that the travelling potential point on the vernier dial can change its potential by the volt drop across one stud

of the main dial when moving from one end to the other of the vernier dial, which can be subdivided into ten or one hundred or any number of equal steps.

It will be evident, however, from Fig. 7 that the potential of the travelling point when at the zero position B will not be the same potential as at the zero point on the main dial, but will be at some higher potential due to the unavoidable resistance of the contact and lead connecting the vernier dial with the main dial. The volt drop between A and B cannot therefore be equal to that between C and D. It is essential, however, that the volt drop between A and B should be exactly equal to the volt drop upon one stud of the main dial so that

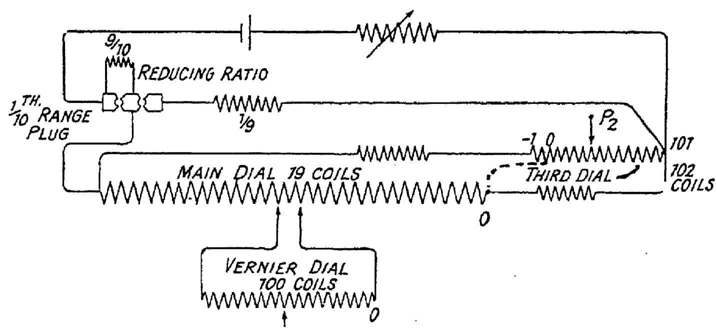


FIG. 9.—Three-dial vernier potentiometer with negative zero.

in moving the vernier dial along the main dial the increments of potential shall agree with that given when the travelling potential point moves from B to A on the vernier dial.

If R = the resistance of each coil of the main dial

r = the resistance of each contact and lead to the vernier dial

R_1 = the resistance of the vernier dial between the two extreme potential points A and B

I = the current in the potentiometer circuit then the current in the vernier dial will be $\dfrac{2R}{R_1 + 2r + 2R}I$. The potential difference between A and B will be $\dfrac{2RR_1}{R_1 + 2r + 2R}I$. The

volt drop on each unshunted coil of the main dial will be IR and since the potential difference between A and B must equal that on the unshunted coils, $\frac{2RR_1}{R_1 + 2r + 2R}I = IR$

whence $$R_1 = 2R + 2r.$$

Thus in order to make the volt drop between A and B of correct value, the total resistance of the vernier dial must be increased above its nominal resistance of twice the main coil resistance, by twice the switch and lead resistance, thus making up the total resistance of the shunting circuit to the value $2R + 4r$. The current in the vernier dial is then $\frac{2R}{4(R + r)}I$ and the volt drop between the points A and B is

$$\frac{2R \,.\, 2(R + r)}{4(R + r)}I = IR$$

as is required.

The volt drop between the points C and D will not be the same as that upon an unshunted coil, but will be higher. This is however of no importance as it will be of constant value and is not in the potential measuring circuit. As the vernier dial is moved, the increment will be that of the volt drop upon an unshunted coil. As long as R, r and R_1 in the above ratio are constant at all positions their absolute resistance values are quite immaterial.

If the contact resistances vary there will be a variation in the potential of the travelling point. This will affect the accuracy to a more or less serious degree according to the position of the contact which varies in resistance. Normally the potential difference between B and D (or A and C) is $2RrI/4(R + r)$. If a change δr in r takes place at D the potential difference between B and D becomes

$$2R(r + \delta r)I/\{4(R + r) + \delta r\}$$

and if δr be neglected in the denominator of this expression, the change of potential at B is $2R\delta rI/4(R + r)$ and further, as r is usually small compared with R, this is approximately $\delta r/2$. Since B is the zero position on the vernier dial, this means that there is a shift of the point of zero potential. If the change takes place at C, the potential difference between

B and D becomes $2\text{R}r\text{I}/4(\text{R} + r) + \delta r$, which is very nearly equal to the normal value for this potential difference, and hence there is no appreciable zero shift. In either case there is a change in the voltage drop between the ends A and B of the vernier dial, which is given by

$$\text{RI}\left(1 - \frac{1}{1 + \delta r/4(\text{R} + r)}\right).$$

It will be seen from the foregoing that the effect of changes in the contact resistance between D and B is much more serious than between C and A. The former causes a zero shift whilst the latter causes only a very small change in the voltage drop between A and B.

In illustration of the above, the following typical figures are given:

If $\text{R} = 10$ ohms $r = 0{\cdot}01$ ohm and the change in r, δr, is 0·001 ohm occurring at D, the zero shift will be 0·0005 I and the change in potential between A and B is $(10 - 10/1{\cdot}000025)\,\text{I}$ which is approximately 0·000025 I.

The volt drop in the coil R would not exceed 0·1 volt in a high-range potentiometer giving a current of 0·01 amp. and a zero shift of 5 microvolts. For a low-range potentiometer the corresponding voltage drop would be 0·01 volt, giving a zero shift of 0·5 microvolt. It should be noted that this figure of error is the maximum which occurs at the travelling potential point, and would occur at B. If the change in contact resistance were at C instead of D, the variation of the voltage drop from one end of the vernier dial to the other will be 25 parts in one million.

A further example of the vernier type of potentiometer is given in Fig. 10. This potentiometer has been specially designed to meet the particular requirements which have been called for by the Electricity Supply (Meters) Act, 1936, and subsequent regulations. It has a main dial giving a volt drop of 1·5 volts in 15 steps of 0·1 volt. The second dial subdivides each step of the main dial into 100 parts, and the third dial each of these into 10 parts so that the 1-volt setting is subdivided into 10,000 parts.

A special feature of the circuit is the arrangement of the

second or vernier dial. This is not equal in resistance to two studs of the main dial as in the usual Varley vernier arrangement, but has a considerably higher resistance. The object of this is to eliminate the effect of contact resistance in the circuit and to secure a very low zero residual voltage.

A separate circuit is provided for the standard cell balance so that the potentiometer dials do not have to be set when balancing the standard cell. This standard cell circuit provides for a temperature range from 10° to 30° C.

Four pairs of test terminals are available so that four separ-

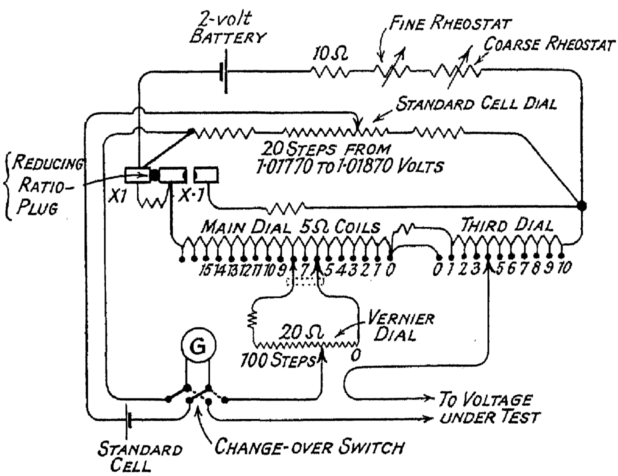


FIG. 10.—Diagram of connexions for E.S.M. precision potentiometer.

ate circuits can be connected to the potentiometer and selected as required for measurement. This avoids changing shunts when a wide range of current has to be measured. The instrument is illustrated in Fig. 11. It has two ranges 1·5 volts subdivided to 100 microvolts on the top range and 0·15 volt subdivided to 10 microvolts on the lower range.

The Substitution Circuit.—Another method of subdividing is by use of a constant resistance circuit made up of two resistance boxes in series mechanically coupled so that as one increases the other decreases and keeps the total resistance constant. This is the principle used in what is commonly called

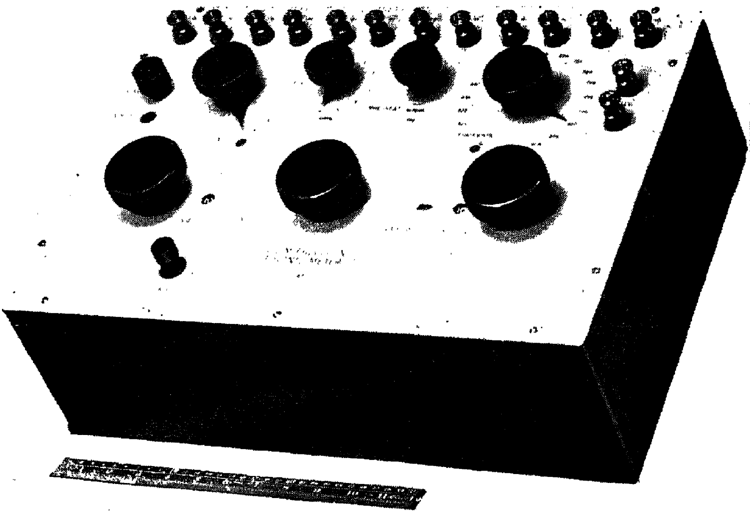


FIG. 11.—E.S.M. precision potentiometer.
(H. Tinsley & Co.)

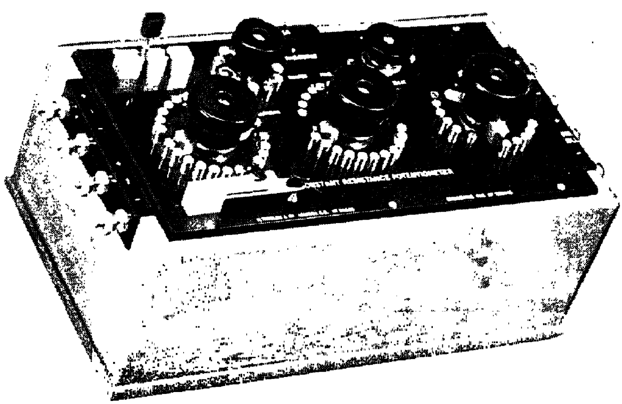


FIG. 15.—Constant resistance potentiometer.

To face p. 20]

the Feussner type of potentiometer, but the substitution principle is really due to Rayleigh and the mechanically coupled dials to Edward Weston.[3] Fig. 12 shows the scheme.

The resistance in the battery circuit remains constant at all positions of the switches, because the resistances $R_1'R_2'R_3'$ in the lower portion decrease as those in the resistances R_1R_2 and R_3 respectively increase, owing to the switch arms being mechanically coupled. The voltage at the potential points is proportionate to the resistance between them, since the current flowing round the circuit remains constant. The two dials R_4 and R_5 are potential dials, the potential points travelling from one end to the other without any alteration to the resistance in the battery circuit.

The dial resistances are usually in decade values, so that

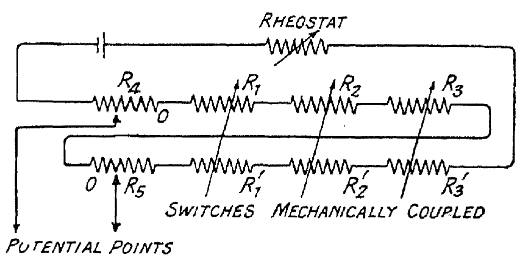


FIG. 12.—Substitution circuit.

each dial subdivides each step of the one above into 10 equal parts. The voltage between the potential points will be zero when the resistances $R_1 + R_2 + R_3 + R_4 + R_5$ are zero, and will increase as these values are increased. In practice true zero cannot be reached because of the residual resistance, but this can be included in the reading of the lowest dial. The current is standardized by setting the dials to the standard cell value and adjusting the rheostat until the standard cell is balanced, in the usual way. It will be obvious that this method of subdivision can be extended indefinitely by increasing the number of substitution dials.

The chief objection to this type of circuit is the number of current-carrying contacts in the circuit which are therefore liable to affect the potentials. To reduce the effect of the

contacts, a relatively high-resistance circuit must be used, and this reduces the sensitivity of the galvanometer when balancing.

In these highly divided potentiometers special precautions are necessary to enable use to be made of the subdivisions. One of the chief difficulties is the thermo-electric potentials which may be set up at the contacts of the potentiometer and which may be larger than the lowest count of the instrument.

When the switches are operated, a certain amount of heat is produced. If the contacts are of dissimilar metals this will cause a thermo-electric e.m.f. to be set up which may give a false value to the voltage acting in the galvanometer circuit. This effect is avoided by using exactly similar metal for both the sliding contact brush and the surface upon which the brush slides. Gold-silver alloy is one of the best materials, but care must be taken to use the same sample for both contact faces. Different samples exhibit thermo-electric differences. By suitable design in this way the thermal e.m.f.s can be reduced to about 0·2 microvolt even with the most vigorous operation of the switches.

The resistance coils of the potentiometer must be accurately adjusted and maintain their resistance values with great precision, and further they must have a small temperature coefficient so that non-uniform temperature throughout the instrument will not affect its precision. In addition it is of the greatest importance that there should be no thermo-electric effects between the resistance coils and the switches and leads, which will almost inevitably be of copper or brass.

There is only one material available which fulfils these requirements: this is the copper-manganese alloy known as manganin.[4–5] The composition of manganin is given as 84 per cent. of copper, 12 per cent. of manganese and 4 per cent. of nickel, but these proportions have been the subject of much research. Its temperature coefficient at about 20° C. in selected samples is less than 0·002 per cent. per degree C. and is usually positive. Its thermo-electric effect against copper is about 1 microvolt per degree C. Its resistivity is about 40 microhms per centimetre[3] and its permanence of resistance when suitably annealed is of a high order. In the author's experience the only reliable commercial source of supply is Germany. A good

material under the name of Therlo can be obtained from the United States of America, but it is much harder. The English made material is no longer marketed, and its performance was inferior when it was available. The properties of manganin are much affected by mechanical stresses and by heat such as occurs in hard soldering copper ends to small potentiometer coils. The coil must be thoroughly cleaned and annealed after winding and soldering, if the resistance is to remain permanent. Only a very thin layer of varnish should be used.

In connexion with temperature measurements by thermo couples, it is sometimes necessary to measure voltages down to hundredths of a microvolt. This calls for very special precautions in potentiometer design. One of the best examples

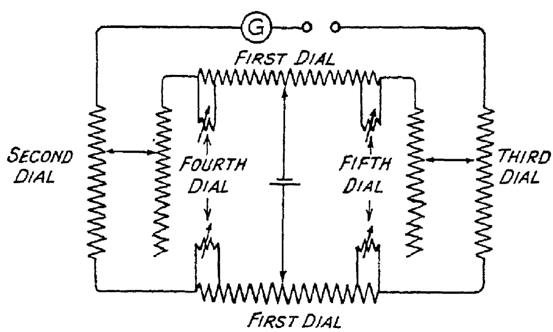


FIG. 13.—Diesselhorst type of potentiometer, simplified circuit.

for this class of measurement is the Diesselhorst type of potentiometer as shown in Fig. 13. In this instrument the effects of thermo-electric potentials set up in the instrument by the movement of the switches during the process of balancing are eliminated from the galvanometer circuit by the very ingenious arrangement of the circuit. The full theory is given elsewhere.[6] There are no switch contacts directly in the galvanometer circuit, which resembles a bridge with the battery across one diagonal, and the other diagonal forming the potential points. The potential between these points is varied by altering the four arms of the bridge in such a way that the resistance between the battery points is always the same, and the switching occurs only in the closed bridge circuit. This means roughly that

any thermo-electric e.m.f.s set up on the switches are merely added to the battery voltage, and therefore can produce only a few parts in a million change in the value of potential, which is quite negligible, as the total voltage being measured in these circumstances is never more than a hundred microvolts.

The fine adjustment fourth and fifth dials are graded shunts of a fairly high resistance upon 1-ohm coils in the bridge arms. Thermo-electric potentials set up in these switches act therefore in a closed circuit of fairly high resistance and can produce only a very small effect upon the potential drop upon the 1-ohm coil. The result is that the worst effect which can appear at the potential terminals is only about 1 per cent. of the actual thermo-electric potential set up in any switch. The principles used in this instrument are worth remembering where problems of making small potential changes are met.

The Constant Resistance or Deflexional Potentiometer.—The deflexional potentiometer is so called because the deflexion on the galvanometer is used to supplement the readings of the dials. With this type of potentiometer an exact balance is not obtained. The unknown voltage is nearly balanced, and

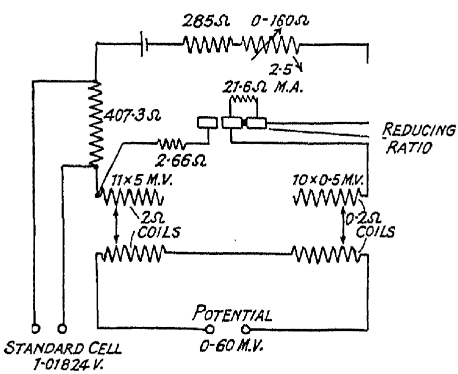


FIG. 14.—Constant resistance potentiometer.

the residue of unbalance read off upon the galvanometer which is arranged to be directly calibrated in volts or microvolts in terms of the last dial.

The essential requirement of the deflexional potentiometer is that the resistance of the galvanometer circuit should be

unchanged with any setting of the dials. In these circumstances the galvanometer current will be directly proportional to the out-of-balance voltage. Several schemes have been devised for such instruments.[7–8]

The circuit of such an instrument without the galvanometer is shown in Fig. 14 and the instrument illustrated in Fig. 15.

Fig. 16 shows the schematic arrangement for a 3-dial instrument specially designed for thermo-couple work. The maximum range is 60 millivolts and each step of the last dial is 50 microvolts. This degree of unbalance is arranged to give a deflexion of 50 millimetres to the galvanometer, so that each millimetre represents 1 microvolt.

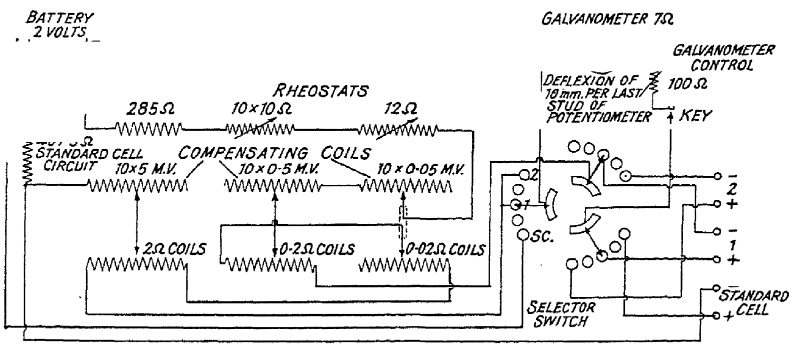


FIG. 16.—Diagram of connexions for constant resistance deflexion potentiometer.

All the contacts and terminals are of copper to reduce thermo-electric effects.

The constancy of the resistance between the potential points of the potentiometer is achieved by the compensating coils which are introduced at each step as the switches are moved. The alteration in resistance due to the third dial is only 0·250 and is negligible. The advantage of this type of potentiometer is the speed with which a reading can be taken ; it is particularly applicable to the measurement of slowly changing voltages such as occur in thermo-couples taking cooling curves, where it is desirable to follow the time-contour of the curve.

Photo-electric Potentiometer.—A new type of potentiometer which may be grouped under this heading has recently

been introduced by the Weston Electrical Instrument Co. In this instrument a slightly different application of the potentiometer principle is employed. The potentiometers so far described are essentially constant current instruments in which a variation of voltage is obtained by the selection of different values of resistance. In this new instrument the resistance of the potentiometer remains constant and the unknown voltage is balanced by alteration of the current through this resistance until the potential drop in it is equal and in opposition to the unknown voltage.

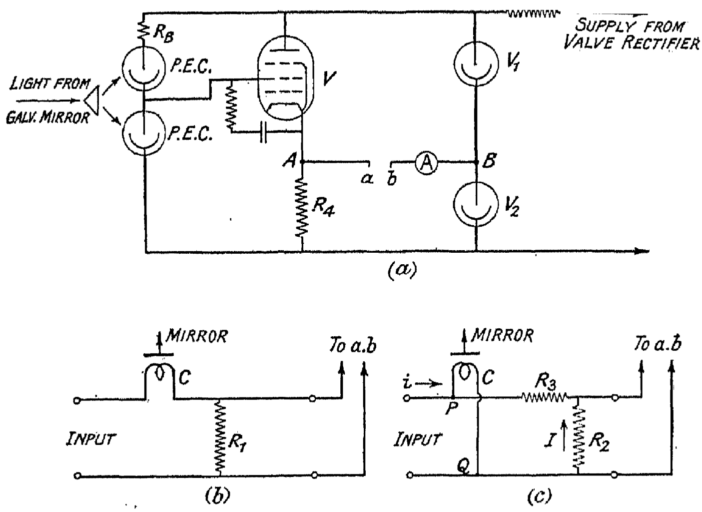


FIG. 17.—Photo-electric potentiometer.

Since the variation in current is a measure of the unknown voltage, an ordinary indicating instrument can be used to give directly the value of the unknown. Further, the circuit arrangement is such that the current variation required for balancing is made automatically, with the result that the instrument gives rapid indications.

The circuit is shown in Fig. 17 (*a*). The valve V, resistance R_4 and the two voltage regulator tubes V_1 and V_2 may be considered as a four-arm bridge network. The resistance R_8 in series with the photo-electric cells when illuminated may be considered as a potential divider by means of which a variable

bias may be applied to the valve. If an indicating instrument (a milliammeter) is connected across the "galvanometer" arm of the "bridge," the potentials and currents in the circuit will adjust themselves so that no current flows in the milliammeter. This may be seen by consideration of the components of the bridge. If the point B assumes a higher potential than the point A, current will flow from B to A. This necessitates a higher potential across V_1 than across V_2 and a greater current in V_1 than in V_2. By the characteristics of these tubes the resistance of V_1 increases, thereby tending to reduce the current in V_1 at the same time as the current in V_2 similarly tends to increase, thus lowering the potential at the point B. With B at a higher potential than A, the current in R_4 is greater than that in V. The potential drop in R_4, therefore, increases causing in effect a lowering of the cathode potential of V. This reduces the bias, increases the current in V, and thus raises the potential of the point A. The action is cumulative and virtually instantaneous, so that A and B assume the same potential. It will only take place, however, provided that the grid of the valve V is held at such a potential that the valve is working on the straight part of its characteristic. The potential divider circuit is designed so that with equal illumination of the two photo-electric cells this condition is fulfilled.

If now the unknown voltage is injected between the points A and B, considerations such as the above will show that the current in the valve will either reduce to zero or assume a very large value according to the polarity of the injected voltage. To control the valve current the circuit shown in Fig. 17 (*b*) is connected to the points *a* and *b*. The unknown voltage is applied to the terminals marked "Input."

On application of the unknown voltage a current due to the unknown passes through the coil *c* of the galvanometer and the standard resistance R_1. The galvanometer coil is deflected and the mirror mounted on its axis causes the beam of light reflected from it to produce a differential illumination of the photo-electric cells. The beam passes through a system of prisms arranged so that deflexion of the beam causes an increase of illumination on one cell and a decrease on the other. The

conductivity of the cells thus alters and consequently also the bias of the valve.

Due to the introduction of the extraneous voltage the " bridge " circuit is unbalanced and consequently current flows through R_1 and the milliammeter. Suitable arrangement of the polarity of the injected voltage will ensure that this latter current is in opposition to the former. If the two are not equal, current will be taken from the unknown source and thus operate the galvanometer coil which in turn modifies the grid bias valve current and the current through R_1. This modification will continue until the coil which has no control torque remains at rest, and it can obviously do this only when the voltage drop due to the current through R_1 from the bridge circuit exactly counterbalances the applied voltage. The current then flowing is then directly proportional to the unknown voltage, and the milliammeter suitably calibrated indicates the unknown directly.

The actions described in some detail here take place almost instantaneously, and thus the instrument provides a rapid indicator potentiometer of virtually infinite impedance.

The range of voltage which can be measured is determined by the value of the standard resistance R_1 employed, for if the indicating instrument gives full-scale deflexion with 10 mA., a resistance of 10 ohms permits a voltage measurement up to 0·1 volt to be made, whilst with $R_1 = 10{,}000$ ohms voltages up to 100 volts can be made.

For the measurement of current the circuit of Fig. 17 (*c*) is employed. The condition of balance is, as before, that no current flows through the galvanometer. For this to occur the potential difference across PQ must be zero. With the symbols as shown in the diagram, where i is the unknown current,

$$i(R_2 + R_3) = IR_2$$

whence $$i = IR_2/(R_2 + R_3).$$

Suitable choice of $R_2 + R_3$ permits measurement of current over a wide range.

It is claimed that under the best possible conditions indications may be obtained of voltage of the order of 10^{-5} volt

and current of the order of 2×10^{-8} ampere. This minimum of response represents the maximum of error within the instrument regardless of full-scale range, so that for the higher ranges the precision of measurement is limited by the precision of the indicating instrument.

Precautions in Precise Measurement.—In precise d.c. measurements, the chief difficulties arise from thermo-electric effects in the case of low voltages, and leakage in the case of high-voltage measurements.

Thermo-electric effects should be reduced as far as possible by suitably constructed apparatus. For example, resistance standards used for current measurements should be free from dissimilar metals at the terminals and manganin should be used for the resistance unit. Nickel-plated terminals are a fruitful source of thermal effects and for this reason pure copper should be used in apparatus for low-voltage work. The galvanometer may also exhibit thermo-electric effects at the various junctions between the suspension strips, coil and terminals, and may have to be protected from temperature changes to obtain steady results. A double reversing key can be used to reverse the galvanometer and the battery simultaneously. This device is almost essential for measurements of very low voltages. Two readings are then taken and if the thermo-effects are unchanged the mean of the two observations will give the correct value, but it is essential that the reversing key is itself quite free from effects. Mercury contacts are often employed for this purpose, but pure copper contacts will give very good results if properly designed.

Chemical Action in the Leads.—Occasionally trouble is experienced due to chemical action in the wiring introducing extraneous potentials into the circuit. This has been observed when rubber-covered galvanometer and potential leads have been employed. In situations liable to chemical contamination, it is best to support the external wiring clear of all contacts between terminal and terminal, then electrolytic action cannot affect the circuits.

High-voltage Measurements.—In high-voltage measurements the aim should be to keep the potentiometer and the circuit upon which the volt drop is being measured at earth

potential if possible. This can be arranged by connecting the volt ratio box (used to obtain a convenient fraction of the high voltage as described later) with the subdivided portion on the "earth" side. Where this cannot be done it may be necessary to employ special guard circuits to prevent leakage currents getting into the galvanometer.

Fig. 18 shows the potentiometer arranged with a guard circuit to by-pass leakage currents.

At other times the potentiometer must be provided with a complete screen raised to the same potential as the point of measurement by means of an auxiliary potential divider and

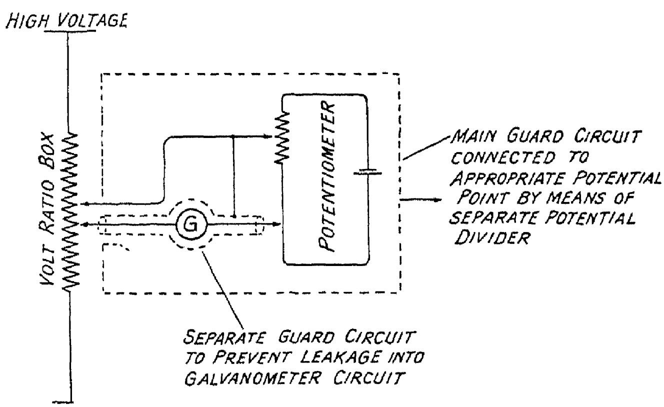


Fig. 18.—Potentiometer with guard circuit.

special protection provided for the operator, in the form of an outer metal case insulated from the screen and earthed with extended handles to the switches with earthing sleeves to prevent leakage up the handles.

In the measurement of very high-resistance circuits such as photo-electric cells and glass electrodes in hydrogen ionization work, similar precautions are necessary to prevent leakage currents getting into the galvanometer circuit. The general principle is to keep the measuring circuit as near to earth potential as possible and provide guard circuits to by-pass the leakage currents back to the battery without going through the galvanometer.

Fig. 19 shows such a circuit, where it will be seen that any leakage will be by-passed from the measuring circuit.

Very low-voltage measurements require very sensitive low-resistance galvanometers, and potentiometers with low-resistance circuits are essential so that the small potential differences can send as much current round the galvanometer circuit as possible.

The presence of spurious effects can usually be detected by disconnecting the current supply to the circuit under test on one side only.

The potentiometer balance should then be zero, since no current should be flowing in the circuit under test. If the balance is not zero the source of the extraneous e.m.f. must

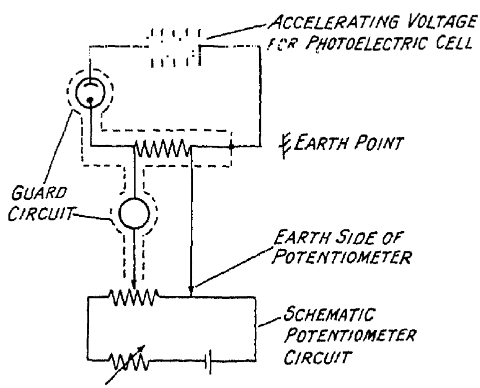


FIG. 19.—Prevention of leakage in high-resistance circuit.

be located. If disconnecting the supply on both sides removes the effect, it is probably due to leakage. If a false zero still remains it is due to thermo-electric or other stray effects within the circuit itself.

The potentiometer is most readily checked for false readings by short-circuiting its potential terminals and observing the zero balance.

The galvanometer leads should be reversed in order to test for thermo-electric effects in this part of the circuit. If a galvanometer shunt is employed, the connexions to this should be reversed also to detect any alteration in the galvanometer zero which often occurs due to effects in the shunt.

If the galvanometer spot wanders when the potentiometer terminals are open-circuited, it indicates some form of leakage or polarization in the circuit. Condensation on an ebonite panel of a potentiometer which has been exposed to sunlight often produces this effect. Sulphuric acid is formed on the ebonite surface by exposure to sunlight and strong electrolytic action will be set up in the circuit, which will make the galvanometer very unsteady. To remove this effect the ebonite should be washed with distilled water and carefully dried. Turpentine on a cotton-wool pad can then be used to clean the surface without spoiling the lacquer of the metal parts. The effect may occur in any ebonite insulation and is not infrequent at the terminals of the galvanometer itself. A simple test of an ebonite surface for the formation of sulphuric acid is to touch it with the tongue. The acid taste is strongly discernible on a deteriorated surface. It is important, therefore, to exclude, so far as practicable, all sunlight from ebonite surfaces, in order to preserve their insulation inert. Systematic open-circuiting, short-circuiting and reversals are the simple means of checking and locating false effects.

Unless both potentiometer and the circuit under test are very steady, great precision of measurement cannot be obtained. There is usually no difficulty in obtaining great steadiness in the potentiometer if an accumulator of ample capacity is used and left connected to the potentiometer for several hours before commencing work. In the case of the circuit under test, the same conditions are requisite, and if the current required is large, a very large accumulator may be necessary to maintain a steady current. By systematic observation, however, with readings taken at uniform intervals, it is possible to obtain high accuracy although the current in the test circuit may be steadily falling. This is largely a matter of practice and in measuring currents of many hundreds of amperes it is almost inevitable. Tests under such conditions are much facilitated if the galvanometer has a short period and good damping so that it responds to changes of adjustment as quickly as they are made.

Independent Standard Cell Balancing Circuits.—In the simple potentiometer circuit the dials are set to the value of

the standard cell and the current through the potentiometer adjusted until the latter correctly balances the standard cell. Although this is the most accurate method of standardizing any potentiometer since it is a direct check of the instrument setting, it is often more convenient to be able to standardize the potentiometer against the standard cell without altering the dials. For example, when a voltage has been balanced it is convenient to be able to check quickly the standardization of the potentiometer without altering the dial setting of the balance. To enable this to be done, an appropriate portion of the potentiometer resistance must be tapped, and a selector switch so arranged that the galvanometer and standard cell can be connected across this portion of the voltage gradient of the potentiometer.

In some cases ten studs of the main dial and a small additional resistance are used to give the required volt drop, but when a vernier dial is used this method may be unsatisfactory because the vernier dial may be shunting part of the circuit upon which the volt drop is being used to balance the standard cell; such an arrangement is therefore dependent upon the dial setting, which is just what it is desired to avoid. A further objection arises when a reducing ratio is provided for lowering the total range of the potentiometer dials, and in this case it is preferable to standardize the total current through the potentiometer by an entirely separate circuit for the standard cell. Such an arrangement is shown in Figs. 6, 10 and 14 and consists of a separate parallel circuit across the whole potentiometer with an appropriate portion taken to the standard cell and galvanometer. The necessary selector switch is omitted for simplification.

In a precise potentiometer it is necessary to vary the tapping point to which the standard cell is joined to allow for the temperature coefficient of the standard cell. This is arranged by inserting a switch dial so that the tapping point can be varied to suit the value of the standard cell. This should cover a range of temperature from 10 to 30 degrees C. corresponding to a voltage range of 1·01854 to 1·01777.

It is convenient to have a selector switch by means of which a number of measuring circuits can be connected to the

potentiometer, so that more than one external voltage can be measured in quick succession.

It is also essential that the rheostats should be adequate in range and extremely fine in adjustment as it is necessary to control the current in a high precision potentiometer circuit to a few parts in a million.

Determining the Internal Resistance of the Standard Cell.—The internal resistance of the standard cell directly affects the precision with which it can be balanced. A good cell should not exceed 2,000 ohms internal resistance and preferably be less. Its value can be readily estimated from the sensitivity of the galvanometer or can be measured by the halving principle ; having balanced the standard cell, unbalance the potentiometer by some small value of voltage which gives a few centimetres deflexion to the galvanometer. Now insert into the galvanometer circuit a variable resistance and adjust this until the galvanometer deflexion is halved. By the previous theorem the total circuit resistance must have been doubled to halve the out-of-balance current. If r_c is the internal resistance of standard cell and r_a the added resistance to halve the deflexion it follows that

$$r + r_c + r_g = r_a$$
$$\therefore\ r_c = r_a - r - r_g$$

where r is the resistance of the potentiometer between the tapping points as explained in Chapter II.

This test should be carried out as quickly as possible to avoid taking current from the standard cell for more than a few seconds.

A very simple method of measuring the internal resistance of a pair of standard cells is to connect them in series with positive to positive or negative to negative so that their e.m.f.s. are opposed, and to measure the resistance of the two cells upon a Wheatstone bridge.

NOTES AND BIBLIOGRAPHY.

[1] Varley vernier dial: Cromwell F. Varley introduced considerable improvement to the divider patented by Thompson and Jenkin in 1860 (Brit. Pat. 2047) ; see *Rep. Brit. Ass.*, 1866. Notices and Abstr., 14.

[2] Gall, Douglas C.: "Recent Improvement in Precision Potentiometers," *J. Sci. Instrum.*, 1935, xii, 284–5.

[3] WESTON, EDWARD, 1892, U.S. Patent 480893. See also WOLFF, OTTO, " Eine neue Ausführungsform des Feussner'schen Kompensationsapparates," *Z. InstrumKde,* 1901, xxi, 227–31.

[4] Manganin: This alloy has been developed in the Physikalisch-Technische Rechsanstalt by Karl Feussner and Stephan Lindeck. See *Wiss. Abh. phys.-tech. Reichsanst., Berl.,* 1895, ii, 503–41; also

[5] LINDECK, S.: " On Wire Standards of Electrical Resistance," *Rep. Brit. Ass.,* 1892, 139–46.

[6] Diesselhorst potentiometer: For full description and theory see DIESSELHORST, H. J., " Thermokraftfreier Kompensationsapparat mit fünf Dekaden und Konstantem Kleinen Widerstand," *Z. InstrumKde,* 1908, xxviii, 1–13.

[7] CARPENTER and STANSFIELD: " Deflection Potentiometer," *Phil. Mag.,* xlvi, 59.

[8] BROOKS, H. B.: " A Deflection Potentiometer for Voltmeter Testing," *Bull. Bur. of Standards,* iv, No. 2; also viii, No. 2.

CHAPTER IV.

USES OF THE D.C. POTENTIOMETER.

The d.c. potentiometer is an instrument capable of use for the determination of voltages with a high degree of precision, and in consequence offers itself as a suitable measuring device whenever high accuracy is required in a voltage measurement. When the voltage to be measured is greater than the normal potential drop in the potentiometer which is of the order of 1·5 volts, it is necessary to make some provision in the test circuit for the measurement of voltages in excess of 1·5 volts. The apparatus used for this purpose is the volt ratio box. When the potentiometer is employed to measure the current in a circuit, use is made of the fact that the potential difference between the ends of a resistance is proportional to the current passing through the resistance. Thus if a known resistance is included in the test circuit to carry the current to be measured, and the voltage across this resistance is measured on the potentiometer, the current is determined from the ratio of the potential difference and the known resistance. Where the value of a current is required with high precision it is obvious that the value of the resistance used must be accurately known, and consequently it is necessary to use standard resistances. These are frequently referred to as shunts.

The volt ratio box and the standard shunt are essential accessories for use with the potentiometer, and will be considered further as a necessary introduction to the uses of the potentiometer.

The Volt Ratio Box.—The volt ratio box [1] consists of a resistance tapped at a suitable point such that the resistance between one end and the tapping point is a convenient fraction of the total resistance. When a voltage is applied across the resistance, that across the tapped section is then this fraction

of the whole. For example, in measuring 100 volts, a resistance of 5,000 ohms tapped at 50 ohms would give a ratio of 1 to 100, so that the potential measured across the 50-ohms section would be one-hundredth of the voltage across the complete resistance, that is, 1 volt. Where it is necessary to measure the very high voltages met with in a.c. distribution systems and the like, special designs and precautions must be adopted. These will be dealt with in a later chapter.

Standard Resistances.—Standard resistances [2] for use with potentiometers should preferably be of the four-terminal type, that is with two current terminals and two potential terminals. With any but resistances of 10 ohms and upward four terminals are essential if a high degree of precision is required, because the end of the resistance becomes indefinite, if only two terminals are used. With a four-terminal resistance, the resistance is defined as the ratio of the volt drop between one pair of terminals to the current passing through the other pair. It may be noted that this value is reversible, so far as the proper current and potential terminals are concerned, but not interchangeable with improper pairs of terminals. A set of four terminal resistance standards is an essential adjunct to the potentiometer. They can be constructed to measure from the smallest to the largest current. The type of construction used by the National Physical Laboratory consists of a number of bare manganin wires hard soldered into copper bars forming a grid. Several grids so formed are soft soldered into massive end castings of brass or copper, which form the terminals. Thin sheet material is more liable to rolling flaws than wire, and its use for resistance standards is, therefore, less reliable although its cooling surface is greater. The current terminals are large bolts in the end casting capable of accommodating the heavy leads necessary for large currents. The potential terminals are smaller and located close to the manganin wires. Care is necessary in the design of low resistances to ensure adequate cooling and to avoid ambiguity in the potential points, with different assembly of the current leads to the terminals. The potential points must be sufficiently remote from the current terminals to ensure a uniform current distribution irrespective of the way in which the current leads are

bolted to the shunt. Fig. 20 shows a low-resistance standard of this type having a resistance of 0·0002 ohm and a current carrying capacity of 750 amperes. Such resistance standards have been constructed for the measurement of currents up to 30,000 amperes.

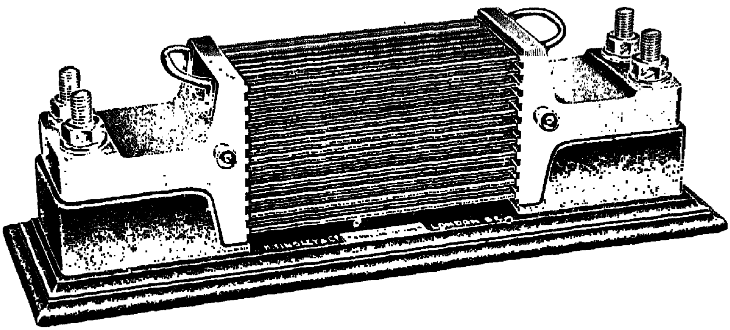


FIG. 20.—Four-terminal resistance standard, 0·0002 ohm, 750 amperes.

Thermo-electric Effect in Shunts.—It is important that resistance standards should be free from internal thermo-electric effects, as these may produce an entirely false potential difference between the potentiometer points. When unsuitable materials are used one end of the shunt acquires a different temperature from the other, due to Peltier effect. This is always accompanied by false potential readings, so that the current is not given by the measured potential.

When manganin and copper are used in the construction of the shunt these effects are negligibly small, but this is not the case with copper and copper-nickel alloys. The presence of thermal effects can be readily observed when they exist, by switching off the current after it has been on for some time and measuring the potential difference upon the resistance. If this is not zero, thermo-electric effects are present.

Measurements with the D.C. Potentiometer.—In illustration of the various purposes for which the potentiometer may be used a few examples of different types of measurements will be given. It is not intended that these should form a comprehensive list of the applications of the potentiometer.

Calibration of a Voltmeter.—Fig. 21 shows the simple

slide-wire potentiometer of Chapter II used for the measurement of the potential difference between the terminals of a voltmeter. The potentiometer is first standardized as explained previously. The voltmeter is supplied with current from a separate battery and the indication of the instrument controlled by a separate rheostat. The position of the tapping point is adjusted on the slide wire until the galvanometer reads zero. Under this condition the voltage drop along the slide wire must be equal to the potential difference at the terminals of the voltmeter, so that the latter is determined from the reading on the slide wire, which in turn is known in terms of

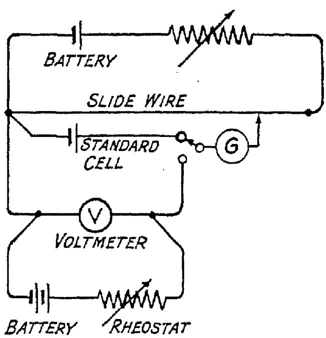


FIG. 21.—The calibration of a voltmeter by means of the potentiometer.

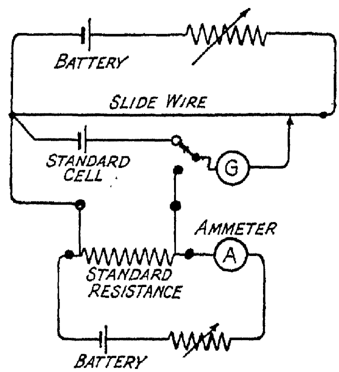


FIG. 22.—The calibration of an ammeter by means of the potentiometer.

the e.m.f. of the standard cell. Thus the true voltage corresponding with any setting of the voltmeter is found.

When the voltages are balanced no current will pass from the voltmeter circuit into the potentiometer circuit or vice versa.

If the range of the voltmeter is greater than the range of the potentiometer, that is if the voltmeter has full-scale deflexion with a voltage greater than about 1·5 volts, a volt ratio box must be used. In this case the volt ratio box is connected across the voltmeter terminals, the voltage across a suitable tapping point being measured on the potentiometer.

Calibration of an Ammeter.—Fig. 22 shows the circuit used for the calibration of an ammeter. A standard resistance is connected in series with the ammeter, and the potential

difference between its terminals measured by means of the previously standardized potentiometer. If the standard resistance is of value 2 ohms and the potentiometer reading at balance is 850 millivolts, the potential difference across the standard resistance is 0·850 volt and the current is therefore

$$0{\cdot}850/2 = 0{\cdot}425 \text{ ampere.}$$

The order of accuracy obtainable in the calibration of instruments by the above method would be insufficient for instruments of substandard accuracy where the reading may be correct to within 1 part in 1,000. In such a case the potentiometer used would necessarily be one of higher precision such as is illustrated in Fig. 11. The illustrations given here merely serve to show the principle of the measurements.

The Calibration of a Wattmeter.—The calibration of a substandard wattmeter is a measurement of considerable importance, because it is one of the basic steps in the calibration of electricity supply meters. Suitable dynamometer type wattmeters will read with equal accuracy on both a.c. and d.c. but are much more easily calibrated upon d.c. To calibrate a wattmeter it is necessary to measure both the current and the voltage, the power will then be given by the product of the two measurements. The measurement is therefore a combination of the two examples given above.

The wattmeter must be set up in a suitable circuit according to its voltage and current range. Generally the voltage is maintained constant across its voltage terminals since this is the usual operating conditions of the wattmeter. The current through the wattmeter current system must be varied from full load to say $\frac{1}{20}$ of full load. Since the current range may be as large as 100 amperes or more it is preferable to provide a separate large-capacity battery for the current circuit, whereas a small-capacity battery is sufficient to supply the pressure system. The total current taken by the voltage system will probably be of the order of 20 to 50 milliamperes, depending upon the values of ohms per volt of the instrument and ohms per volt of the volt ratio box which must be connected across its terminal for the purpose of measuring the voltage. This current can be supplied from a small-capacity "high tension type" of accumulator or from large dry cells. Control of the

voltage can be made by a series rheostat of sufficient resistance to drop the difference between the voltage of the battery and the nominal voltage of the wattmeter. This may be, for example, 110 or 220 volts, in which case volt ratios of 100/1 or 200/1 respectively would be used in the volt ratio box.

The control of the current in the current circuit is an important matter since fine adjustments are necessary. The voltage applied to this circuit need not be more than about 2 volts since the volt drop in the current system of the wattmeter will probably not exceed 1 volt and a maximum volt drop of 1 volt on the shunt in series with it for measuring the current should be sufficient. If a higher volt drop is required the battery

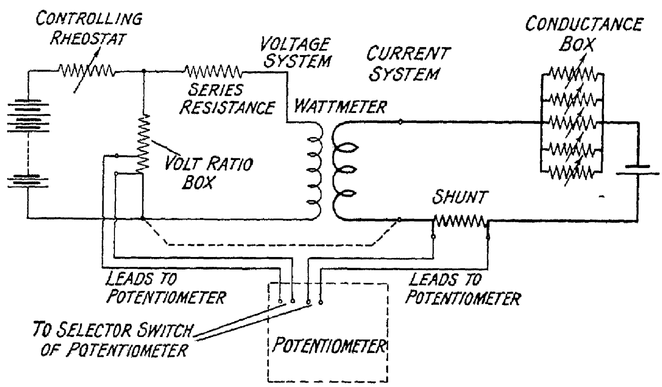


FIG. 23.—*Circuit diagram for calibration of a wattmeter.*

voltage must be increased accordingly, but this is only likely to occur with low-range instruments.

The best method of controlling the current is by means of a conductance box rather than a resistance box or simple series rheostat. The conductance box is dealt with later. By its use very fine steps of current change can be made and held steady. It is useless to attempt accurate potentiometer measurements if the current circuit contains contacts which vary their resistance or resistance undergoing continual change due to heating. This is one of the defects of the carbon rheostat which is therefore not a good device for potentiometer work. The circuit diagram for the complete test is shown in Fig. 23.

It is essential to hold both the current and the voltage steady during each pair of readings. This is best ensured by using batteries of ample capacity and by allowing the circuit to remain on load for at least an hour before taking readings.

If two potentiometers are available, one observer can measure the voltage and control this by adjustment at its correct value while the other measures the current with the second potentiometer by the volt drop on the shunt in the current circuit. An alternative sometimes employed is for the first observer to control the voltage by means of a previously calibrated precision type voltmeter across the voltage terminals of the wattmeter. In either of these methods the second observer has only to make a simple current measurement and is therefore able to give all his attention to this.

Where both voltage and current have to be measured on the same potentiometer the leads from the volt ratio box and the shunt must be brought to the potentiometer. A two-position selector switch for transferring either circuit for measurement is essential to enable a quick changeover to be made. After a preliminary adjustment to the desired values the two measurements should then be made in rapid succession, and the results noted as quickly as possible. If a number of readings are made, with practice it is possible to obtain high accuracy even when the values are running down during the readings. A great aid in this measurement is an extra tapping point on the potentiometer for the voltage circuit. Since the voltage is kept constant, by fixing the voltage tapping on the potentiometer at the required reading the voltage can be checked at once by turning the selector switch, without altering the dial setting. This device is a great time saver.

It is important to maintain both the current and voltage system of the wattmeter at the same potential, otherwise there may be electrostatic forces acting between them which will affect the calibration. This is most easily done by commoning one current to one voltage terminal. In some wattmeters this is done permanently on the instrument. Care must be taken to see that this connexion is not made on the line side of the series resistance of the voltage system of the wattmeter.

A Voltage Controller.—The voltage can be most easily

controlled at a constant value by means of a voltage controller. This consists of a special form of volt ratio box with the tapping point so chosen that the volt drop is equal to the standard cell value when the correct voltage is across the total resistance. The circuit is shown in Fig. 24. A separate galvanometer and standard cell are preferable so that the circuit can be kept quite separate from the potentiometer used for the current measurement. The voltage controller should be provided with a variable tapping point to allow for variation in the standard cell voltage as well as a number of resistance values in the main circuit corresponding to the definite voltage value which has to be controlled; for example, 500, 220 and 110 volts corresponding

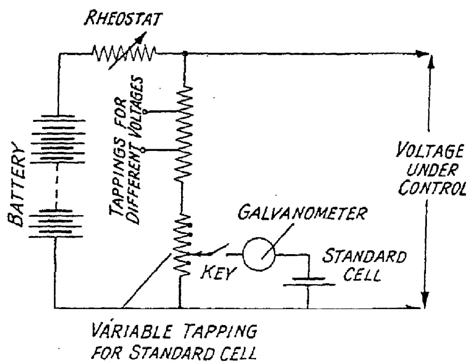


FIG. 24.—Voltage controller for wattmeter calibration.

to resistance values of 50,000, 22,000 and 11,000 ohms respectively, with the standard cell tapping point varying between 101·77 and 101·58 ohms in say 20 steps.

The following Table I gives the measurements involved in the calibration of a Torsion head dynamometer wattmeter. Although principally used for a.c. measurements, the instrument is calibrated with direct current. In order to avoid the use of a high-voltage battery for the pressure circuit, the series resistor is disconnected and a standard 10-ohm resistance substituted. The current in the pressure circuit is then maintained constant at 20 milliamperes by measuring on the potentiometer the volt drop across the 10-ohms resistor. This current corresponds with the full working voltage on any range of the watt-

meter, for the series resistor has a resistance of 50 ohms per volt. The series resistance value is checked by means of a Wheatstone bridge.

The volt drop upon the shunt in series with the current coil serves to determine the wattmeter current, and the required value is obtained by adjustment of the current to give balance at the requisite setting of the potentiometer. Corrections are applied to allow for the difference between the true resistance of the shunt and its nominal value. Thus if the shunt resistance is 0·1 per cent. low, the volt drop upon it must be adjusted to be 0·1 per cent. low in order to obtain the correct current.

The same procedure is adopted for a polyphase wattmeter. In this case interference tests between the two systems are made with full load current in one current system and full load current in the other pressure system. In a perfect instrument there should be no interference. In a good instrument the interference should not exceed the least count of the dial. In order to determine if the d.c. calibration is reliable when the instrument is used with a.c., a zero power factor test upon a.c. is used. An exact phase displacement of 90° between the full load current and voltage supplied to the wattmeter systems is produced. Under these conditions the instrument should read zero. Eddy current or capacitance effects in the instrument which would vitiate its accuracy for a.c. measurements are shown readily by this test.

Wattmeter Calibration.—A typical test of a 10-ampere Single-phase Wattmeter using corrected figures for the Standard resistances in circuit is shown in Table I. Wattmeter current ranges 10–5–2–1 amps. Calibration on 10 amperes range at 20° C. Standard shunt in current circuit nominal resistance 0·1 ohm. Actual resistance 0·0999950 ohm at 20° C. Standard shunt in pressure circuit nominal resistance 10 ohms. Actual resistance 10·001080 ohms at 20° C.

The Comparison of Resistances.—One of the uses of the precision d.c. potentiometer is the comparison of resistances. The potentiometer method is particularly applicable to low resistances such as are used for the measurement of large currents. The method is very simple and the scheme is shown in Fig. 25. The two resistances to be compared are connected in

TABLE I.

POTENTIOMETER SETTINGS.

Current in Current Circuit. Amperes.	Potentiometer Setting (Potential Circuit) No. 1.	Current in Pressure Circuit. Amperes.	Potentiometer Setting (Potential Circuit) No. 2.	Nominal Wattmeter Deflexion.	Actual Wattmeter Deflexion.
				Divisions.	Divisions.
10	0·99995	0·02	0·20002	500	500·2
8	0·79996	0·02	0·20002	400	400·0
6	0·59997	0·02	0·20002	300	299·8
4	0·39998	0·02	0·20002	200	199·8
2	0·19999	0·02	0·20002	100	99·8
1	0·09999	0·02	0·20002	50	50·0

series with a suitable large-capacity accumulator and regulating resistance. The volt drop upon the two resistances is then compared by successive measurements of the volt drop upon the standard resistance and upon the unknown resistance.

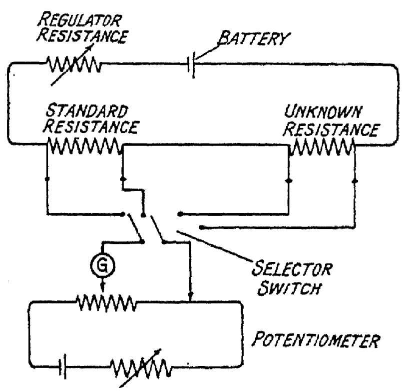


FIG. 25.—Comparison of low-value resistances.

It should be noted that the potentiometer need not be standardized against the standard cell for this comparison and it is an advantage to dispense with this process and use instead the volt drop upon the standard resistance as the standardizing value. Thus, if the potentiometer dials are set at an integral value equal to that of the standard resistance, and the current in either circuit adjusted until the galvanometer balances, the

volt drop then measured upon the unknown resistance will be its true value in terms of the standard resistance. For example, if the standard resistance is known to have a value of 0·010003 ohm, the potentiometer dials would be set to 10003. The current would then be adjusted until the volt drop on the resistance was exactly balanced by the volt drop on the potentiometer dials. The exact value of the volt drop would be quite immaterial so long as it was sufficient to give the desired sensitivity. This would probably necessitate a current of not less than 10 amperes in the resistance or a volt drop of about 0·1 volt.

If the potentiometer is now transferred to the unknown resistance and the volt drop thereon measured, the value of the unknown resistance will be equal to the potentiometer reading.

This method is widely employed in testing the resistance of the copper cores of large cables during manufacture. The control of the first balance is usually made by adjusting the current entirely in the potentiometer circuit, although it might be made in either circuit. For this purpose the normal rheostat of the potentiometer gives insufficient range and a four-dial resistance box is inserted in series with the battery of the potentiometer.

Measurement of Earth Resistivity.—A further example of the use of the potentiometer is in the measurement of earth resistances. Measurements of the resistance of the earth are employed in prospecting for minerals, in investigating the effects of telephone interference from power lines, in determining the probability, or otherwise, of lightning striking certain localities and in various geological problems.

One of the chief difficulties associated with the measurement is that while the resistance of the earth itself is relatively low on account of its enormous cross-section, the resistance of a contact with the earth may be very high. In measuring the resistance between two points on the earth's surface, the earth is therefore treated as a four-terminal resistance, current being passed through two earth contacts and the potential difference measured between two other contacts with the earth. It is usual to space these four contacts at three equal distances because the resistivity of the earth can then be calculated by an empirical rule due to Wenner,[3] which is given below.

Natural Earth Currents.—A further difficulty arises in measuring the voltage between the potential points, due to natural earth currents and to polarization if a metal electrode is in contact with the moist earth. Polarization is overcome by the use of a liquid electrode consisting of a porous pot containing copper-sulphate solution into which a copper rod dips, or a porous pot containing sodium chloride solution into which a silver rod dips. Such electrodes when in contact with the ground do not exhibit a higher degree of polarization than one millivolt or less. These non-polarizing electrodes form the potential points to which the potentiometer is connected. Current is fed into the ground at two copper or steel electrodes driven into the earth. If the voltage between the potential points is measured and found to be E volts when a current of I amperes is flowing in the current circuit, the mutual resistance between the current and potential circuits will be given by $R = E/I$ ohms, and by Wenner's formula: if the electrode spacing is a centimetres, the resistivity of the ground will be $2\pi a R$ ohm-centimetres.

In most localities there are natural earth currents which give a potential difference on the earth's surface. This will vitiate the measurement, but the difficulty can be overcome by the method devised by A. B. Broughton Edge,[4] which involves the use of an auxiliary potentiometer to balance the natural potential differences before making the measurement.

Fig. 26 shows the scheme. The natural earth potential between the potential electrodes is first balanced by means of the auxiliary potentiometer when no current is fed into the ground, and the right-hand potentiometer is set at zero.

The current is then switched on and the value is noted on the milliammeter, and the voltage balanced by means of the right-hand potentiometer. The direction of the current is then reversed and the measurements repeated. Both potentiometers are provided with reversing switches which are omitted from the diagram for simplicity. The mean of the direct and reverse readings eliminates any small polarization occurring in the circuit. In the standard instrument designed by A. B. Broughton Edge the potentiometer is standardized by means of its own galvanometer used as a current-measuring

instrument, and the whole forms a portable self-contained instrument.

It is interesting to note that, theoretically, if the position of current and potential points is interchanged, the same value of R will be obtained.

Temperature Measurements.—The d.c. potentiometer is very widely used for temperature measurements by means of thermo-couples.[5–19] The voltage difference between two junctions of dissimilar metals is an accurate measurement of the

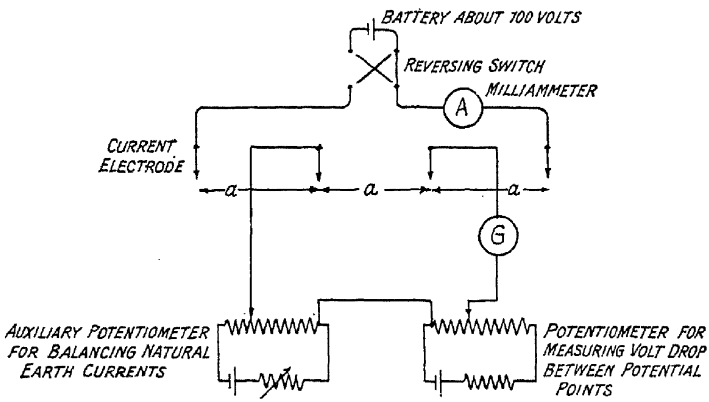


FIG. 26.—Measurement of earth resistivity using an auxiliary potentiometer.

temperature difference. The metals commonly used for low temperatures are copper or iron and nickel-copper alloys. For high temperature and precision measurement platinum and platinum-rhodium alloy are used. With the former, voltages of the order of 60 millivolts may have to be measured, but with the latter the thermo-electric effects are smaller and voltages of less than 100 microvolts often have to be measured with high precision. This necessitates a type of potentiometer which is free from thermo-electric effects, as has been described in Chapter III, and a very sensitive galvanometer.

NOTES AND BIBLIOGRAPHY.

1 "Volt Ratio Box," GLAZEBROOK, *Dictionary of Applied Physics*, ii, 619–20.
2 Standard resistances. For description and bibliography see GLAZEBROOK, *Dictionary of Applied Physics*, ii, 620–2, 700–10.

[3] WENNER, FRANK: "A Method of measuring Earth Resistivity," *Bull. U.S. Bur. Stand.*, 1915–16, xii, 469–78.

[4] EDGE, A. B., and LABY, T. H.: "Principles and Practice of Geographical Prospecting," 1931, Cambridge University Press.

[5] STANSFIELD, ALFRED: "On some Improvements in the Roberts-Austen Recording Pyrometer with Notes on Thermo-electric Pyrometry," *Phil. Mag.*, 1898, (5), xlvi, 59–80.

[6] HARKER, JOHN A.: "A Direct Reading Potentiometer for Thermo-electric Work," *Phil. Mag.*, 1903, (6), vi, 41–6.

[7] LEHFELDT, R. A.: "A Potentiometer for Thermocouple Measurements," *Proc. Phys. Soc.*, 1903, xviii, 469–73.

[8] HAUSRATH, HERBERT: "Die Messung Kleiner Temperaturdifferenzen mit Thermoelementen und ein Kompensationsapparat mit Konstantem Kleinem Kompensationswiderstand bei Konstant bleibenden Hilfsstrom," *Ann. Phys. Lpz.*, 1905, (4), xvii, 735–43.

[9] DIESSELHORST, H. J.: "Über einen Kompensationsapparat mit Kleinem Widerstand," *Z. InstrumKde*, 1906, xxvi, 173–84.

[10] DIESSELHORST, H. J.: "Über thermokraftfreie Kompensationsapparate mit Kleinem Widerstand," *Z. InstrumKde*, 1906, xxvi, 297–305.

[11] WHITE, WALTER P.: "Thermokraftfreie Kompensationsapparate mit Kleinem Widerstand und Konstanter Galvanometerempfindlichkeit," *Z. InstrumKde*, 1907, xxvii, 210–19.

[12] WHITE, WALTER P.: "Potentiometer Installation," *Phys. Rev.*, 1907, xxv, 344–52.

[13] DIESSELHORST, H. J.: "Thermokraftfreier Kompensationsapparat mit funf Dekaden und Konstantem Kleinem Widerstand," *Z. InstrumKde*, 1908, xxviii, 1–13.

[14] WHITE, WALTER P.: "Thermoelement Installations, especially for Calorimetry," *J. Amer. Chem. Soc.*, 1914, xxxvi, 185–68.

[15] WHITE, WALTER P.: "Potentiometers for Thermo-electric Measurements, especially in Calorimetry," *J. Amer. Chem. Soc.*, 1914, xxxvi, 1868–85.

[16] WHITE, WALTER P.: "Einige neue Doppelkompensatoren," *Z. InstrumKde*, 1914, xxxiv, 71–82, 107–13, 142–51.

[17] WHITE, WALTER P.: "A Universal Switch for Thermoelectric Work and other Potential Measurements," *Amer. J. Sci.*, 1916, (4), xli, 307–16.

[18] RANDALL, M., VON BUCHOVSKY, F. R., and RODEBUSH, W. H.: "The Practical Installation of the Double Combination Potentiometer," *J. Amer. Chem. Soc.*, 1916, xxxviii, 1266–70.

[19] WHITE, WALTER P.: "Note on a Universal Switch for delicate Potential Measurements," *Amer. J. Sci.*, 1918, (4), xlvi, 610–12.

CHAPTER V.

GALVANOMETERS.

The detector by means of which the balance point on a potentiometer is determined is an important part of the potentiometer equipment. Various types of detector are used, each with its merits and disadvantages. Certain basic principles in the operation of these detectors are determined by the conditions of use and the circuit of the potentiometer. For instance, since the detector is to be used for the measurement of small voltages, its voltage sensitivity should be high; also to enable rapidity of operation it is desirable that it should respond quickly to the alterations made in the circuit. These points will now be discussed more fully together with the factors influencing the choice of the detector for a particular measurement. In general it may be taken that the detector used in potentiometer work is one of the various types of galvanometer, either with or without some means for amplification of the out-of-balance voltage.

The Moving-Coil Galvanometer.—The length of wire L in the coil of the galvanometer is given by the product of the number of turns N and the mean length of turn l, *i.e.*, $L = Nl$. The number of turns is given by the area of the space available for the winding A, divided by the cross-sectional area a of the wire (including insulation) used. $N = \frac{A}{a}$ whence $a = \frac{A}{N}$. If the coil is of fixed geometric dimensions the length of mean turn and the winding space are constant, then $L \propto N$ and $a \propto \frac{1}{N}$ and hence the resistance of the coil, which is proportional to the length of wire and inversely proportional to its cross-sectional area, is directly proportional to the square of the number of turns. That is $r \propto N^2$ or alternatively $N \propto \sqrt{r}$ where r is the coil resistance.

The galvanometer torque τ is proportional to the strength of the magnetic field H in which the coil swings and the ampere-turns iN of the coil, that is

$$\tau \propto \mathrm{H}i\mathrm{N}$$

so that for a given current

$$\tau \propto \mathrm{H}\sqrt{r}$$

which gives the current sensitivity. If the applied voltage is e then $i = \frac{e}{\mathrm{R}}$ where R includes the coil resistance and the external resistance, and for a given voltage

$$\tau \propto \mathrm{HN/R}$$

and since $\mathrm{N} \propto \sqrt{r}$

$$\tau \propto \mathrm{H}\sqrt{r}/\mathrm{R}$$

which gives the factors determining the voltage sensitivity.

From the point of view of the desirable rapidity of response, the damping of the galvanometer is of great importance. If the galvanometer is overdamped the response is so sluggish that the conditions of balance cannot be found with any certainty, as the true zero cannot be observed. If underdamped, the galvanometer overswings and the process of adjustment becomes very slow due to the time required for the galvanometer to come to rest. The galvanometer should be critically damped or slightly overdamped when the potentiometer key is closed, so that it comes to rest in about one complete swing.

The damping, which is almost entirely electro-magnetic and due to the generated e.m.f. in the coil, is governed by the field strength of the magnet and the total resistance of the circuit. If the field strength is too great, the galvanometer will be overdamped, but if the field strength is too weak, the sensitivity will not be sufficient.

The galvanometer will be critically damped when [1]

$$\mathrm{H}^2 = \frac{\mathrm{PR}\tau}{\pi \mathrm{L}^2 b^2}$$

when $\mathrm{P} \equiv$ natural period
$\mathrm{R} \equiv$ total circuit resistance
$\tau \equiv$ restoring force
$\mathrm{L} \equiv$ total length of conductor forming the coil
$b \equiv$ radius of moving coil

since all of these are constant for a given galvanometer except the resistance it follows, since $r \alpha L^2$

$$\text{that, } H \propto \sqrt{\frac{R}{r}}.$$

Combining the voltage sensitivity and critical damping, the condition for maximum sensitivity with critical damping will require that

$$r \propto \frac{1}{\sqrt{R}} \text{ is a maximum.}$$

It will be evident that this is a maximum only when the galvanometer resistance r and the total circuit resistance R of which it forms part is as small as possible.

Choice of Galvanometer Resistance.—In practice the expression deduced above means that in order to obtain the maximum voltage sensitivity in a potentiometer circuit, the galvanometer should have the lowest possible resistance and the potentiometer circuit should be of as low resistance as possible. The field strength of the galvanometer should then be adjusted to give critical damping.

It is often thought that the galvanometer resistance should match the circuit resistance for maximum sensitivity, but this is not true. If this condition obtains, the power delivered to the galvanometer for a given voltage acting in the circuit will be a maximum, but that is not the condition of maximum voltage sensitivity.

In many cases there is no control of the field strength of the galvanometer magnet. In these cases the galvanometer should have about one-third of the external resistance, as this permits critical damping to be obtained with a reasonably high field strength, thus giving good sensitivity.

It is interesting to note that for maximum current sensitivity R must be a maximum. This means that the galvanometer resistance and the external resistance must be as high as possible. In the latter circumstances, a very high field strength can be used without overdamping, thus increasing the current sensitivity of the galvanometer. This condition is, of course, quite unsuitable for potentiometer work, but applies to such measurements as insulation testing.

Even when the external circuit resistance is high, the highest voltage sensitivity will be obtained with a low-resistance galvanometer, using a high field strength, made possible by the high circuit resistance without overdamping.

A very sensitive galvanometer is of no practical advantage unless its zero is stable, since it becomes impossible to make use of the sensitivity in determining a balance if the zero shifts. The stability of the zero is governed by the strength of the restoring force exercised by the suspension. If this is weakened to increase the sensitivity, the zero may become less stable and may nullify the advantage. If the coil is swinging in a very strong magnetic field the zero may be unstable due to magnetic impurities in the coil becoming magnetized in different orientations when it is deflected from zero. Thus these two things which both increase the galvanometer sensitivity, increase the instability at the same time. In practice a compromise has to be made, and the most efficient galvanometer is one where the compromise has been most effective. It is always more comfortable to work with a short-period galvanometer which follows the adjustments made to the circuit quickly, but the sensitivity of a galvanometer is proportional to the square of the period, so that the sensitivity is reduced to one-quarter if the period is halved. Thus a compromise again becomes necessary between speed of response and sensitivity.

Comparison of Moving-Coil and Moving-Magnet Galvanometers.—The moving-coil galvanometer is a more convenient instrument to use than the moving-magnet galvanometer and is generally preferable because of its freedom from disturbance due to magnetic fields, and the better damping of the moving coil. The limitation in the moving-coil galvanometer is due to its resistance. It is not possible to construct a sensitive moving-coil galvanometer with a resistance of less than say 5 ohms, because of the resistance of the ligaments conveying the current to the coil, which must be very thin so that the coil can swing freely. The most successful type of sensitive low-resistance moving-coil galvanometers employs a very small coil of silver wire to which the current enters by gold-leaf ligaments, the coil being suspended upon a fine quartz fibre. The resistance of the galvanometer limits its voltage

sensitivity. In the case of the moving-magnet galvanometer the resistance can be made extremely low, because the coil is fixed and can be wound with very large wire. When current flows in the fixed coil the tiny suspended magnets move the mirror attached to them which reflects a spot of light on to a scale. Thus for the measurement of very low potentials it may be necessary to have recourse to a moving-magnet type of galvanometer, with its concurrent disadvantages. One or two magnetic screens of high permeability nickel-iron alloy are usually employed to surround the whole galvanometer system to reduce magnetic disturbances, but the magnetic system must first be made very astatic, as magnetic screening is not a complete cure for this. The moving-coil galvanometer can give a sensitivity so that a change of 10^{-8} volt can be observed, while a very good moving-magnet galvanometer will detect 10^{-9} volt. "String" galvanometers, which consist of a single fine wire in a strong magnetic field, are not convenient for general purposes because they require observation by microscope, which is very fatiguing, or by magnified projection, and in addition, their voltage sensitivity is low.

Methods of Amplification.—To obtain sufficient sensitivity for potential measurements in very high-resistance circuits, the method of Drs. Poole and Atkins [2] can be used. This is to interrupt the very small out-of-balance current by means of a clockwork interrupter and then amplify and listen upon telephones. In the case of ionization measurements with glass electrodes, where the circuit may have a resistance of many megohms, an electrometer valve amplifier can be employed. The circuit designed by Drs. Platt and Winfield [3] for this purpose is shown in Fig. 27. The electrometer valve is one of which the grid impedance is extremely high, being of the order of 10^{12} ohms. Its amplification factor is unfortunately small. It does not, however, shunt the high-resistance circuit across which it is used to measure the voltage, thus acting as a detector of almost infinite resistance. The electrometer valve is followed by a second amplifying valve in the anode circuit of which the galvanometer is inserted.

Because of the difficulty of d.c. amplification, the method used is to operate the key slowly by hand and amplify the

pulses of out-of-balance voltage which are impressed upon the grid of the electrometer valve when the key is depressed. A galvanometer with rather special ballistic properties is necessary

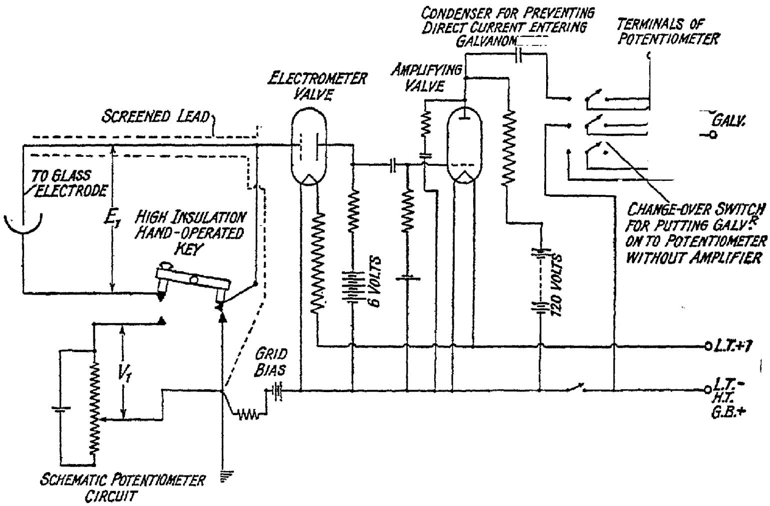


FIG. 27.—Method of amplification of out-of-balance voltage E_1 and V_1 in opposition. When balanced no galvanometer current flows when the key is operated.

to take full advantage of this principle. A sensitivity such that a potential difference of 0·0002 volt through a 600-megohm resistance can be detected, is obtainable by this method. Where a lower sensitivity is sufficient an electrometer [4] can be used without any amplification.

NOTES AND BIBLIOGRAPHY.

[1] LAWS, FRANK A.: "Electrical Measurements," Ist Ed., pp. 38–43, McGraw-Hill Book Company, New York.

[2] POOLE, HORACE H., and ATKINS, W. R. G.: "Further Photo-electric Measurements on the penetration of Light into Sea-water," *J. Mar. Biol. Assoc. U.K.*, 1928, xv, 455–83.

[3] PLATT, BENJAMIN S., and DICKINSON, S.: "The Technique of Glass Electrode Measurement," *Bio-chem. J.*, 1933, xxvii, 1069–77.

[4] Lindemann Electrometer. LINDEMANN, F. A., and KEELEY, T. C.: "A New Form of Electrometer," *Phil. Mag.*, 1924, xlvii, 577–83.

CHAPTER VI.

THE A.C. POTENTIOMETER.

Introduction.—Since the potentiometer is so powerful an instrument for the measurement of steady voltages, its application to corresponding alternating voltage measurements is an obvious extension of its sphere of utility. There are, however, two major difficulties which must be surmounted—the first being the need to balance the phase of the measured potential as well as the magnitude, and the second to standardize the potentiometer, as there can be no alternating standard cell of chemicals for this purpose.

The solution to the first difficulty is found in two ways. One is to rotate the phase of the measuring voltage until it agrees with the unknown, and to adjust its magnitude until it balances the unknown. That is, the potentiometer produces a polar vector of variable phase and magnitude, and is known as a polar potentiometer. The two main instruments of this type are the Franke machine and the Drysdale a.c. potentiometer. The second way of balancing the phase is to use two components of voltage at right angles and adjust their values until the sum of the two components balances the potential to be measured. The main instruments of this type are the Larsen, the Gall, the Pedersen and the Campbell-Larsen a.c. potentiometer. These give the value of the voltage measured in rectangular co-ordinates either directly in volts in the usual symbolic vector form, or the components of the voltages are derived by simple calculations from the circuit components.

The difficulty of standardization is nearly always overcome by using a current-measuring instrument in the potentiometer instead of a standard cell, alternatively use is made of a device which employs the heating effect of a direct current to compare with the heating effect of an alternating current. The

value of the direct current is standardized by means of the standard cell.

There have been many types of a.c. potentiometer described from time to time, but only a limited number have come into general use. A general historical description will be found in Albert Campbell and Ernest C. Childs' "The Measurement of Inductance, Capacity and Frequency," Macmillan, pp. 435–41. In the following pages the circuit and method of use of the most important of these are given.

The Franke Machine.—The Franke machine [1, 2] was the first true a.c. compensator consisting of two alternators mounted in one unit with a means of rotating the stator of one machine with respect to the other. In this way the phase of the voltage of one machine can be varied with respect to the other. The magnitude of the voltage can also be controlled. Although this apparatus performed the function of a potentiometer it was more a machine than an instrument, and the same feature has been employed in many subsequent machines.

The Drysdale A.C. Potentiometer.—The Drysdale a.c. potentiometer [3–6] was first made in 1908 and was the first complete instrument comparable with a d.c. potentiometer. It consists of a phase-shifting transformer [7] which supplies current to a potentiometer. The phase-shifting transformer allows the phase of the current in the potentiometer to be rotated through any angle and the potentiometer provides for variation of the magnitude of the voltage. In this way any unknown voltage can be balanced both for phase and magnitude, and the result is given as a polar vector.

In order to standardize the potentiometer, the instrument is first energized with d.c., just like a d.c. potentiometer, and balanced against the standard cell.

The simplified circuit is shown in Fig. 28. In the potentiometer circuit a Weston dynamometer type milliammeter is inserted giving a full-scale reading with about 55 milliamperes. The resistance of the potentiometer is 2 ohms per stud on the main dial, giving a total resistance of about 40 ohms. From 6 to 8 volts are necessary to provide the 50 milliamperes, which gives the correct volt drop upon the dials, on account of the resistance of the dynamometer.

The dials are set to the standard cell value of 1·01830 volts (at 18° C.) and the rheostat adjusted until the standard cell voltage is balanced. The dynamometer should now indicate 50 milliamperes and it is usual to move the zero adjustment of the dynamometer until the pointer coincides with the fine line on the scale which marks the standardized position.

The Weston dynamometer reads accurately upon both a.c. and d.c. so that if the current through the potentiometer when upon a.c. is adjusted to reproduce exactly this reading, the same effective or r.m.s. value of current must be flowing. If the impedance of the potentiometer coils is the same upon a.c. and d.c. then the effective or r.m.s. value of potential drop

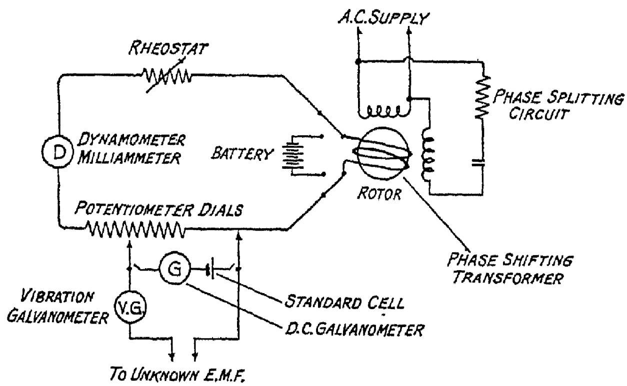


FIG. 28.—Schematic diagram of Drysdale a.c. potentiometer.

upon the potentiometer coils will be the same upon a.c. as upon d.c. when standardized in this way.

The battery is now replaced by the phase-shifting transformer, which feeds a.c. from its rotor into the potentiometer. This current should then be adjusted until it remains constant at the 50 milliamperes value as indicated by the dynamometer, for all positions of the rotor. When these conditions have been fulfilled the instrument is standardized and may be used for the measurement of unknown voltages.

The Phase-Shifting Transformer.—The phase-shifting transformer is the essential part of the Drysdale potentiometer. It consists of a stator having 48 teeth wound with two similar

windings displaced 90° in space with respect to each other. The two windings each produce a linear distribution of flux diametrically across the rotor at right angles to each other. The rotor, which has 36 closed slots, is also wound with two similar windings displaced 90° in space with respect to each other. The linear flux distribution of the stator induces a voltage in the rotor winding which is proportional to the sine of the angle to which the rotor winding is inclined to the stator flux.

Thus if the winding of phase I of the stator is excited by a.c. and the voltage induced in the rotor winding is measured at various angular positions of the rotor, the result will be a sine wave of voltage as shown in Fig. 29.

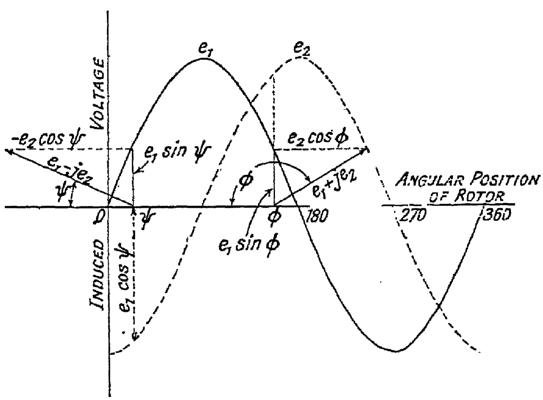


FIG. 29.—Voltage induced in phase-shifting transformer rotor when turned to different angular positions.

When the two stator windings are excited from a two-phase source of equal voltage e_1 and e_2 90° out of phase with each other, then the two sine curves of induced voltage will be added vectorially at 90°. The resultant voltage will be

$$\mathrm{E} = e_1 \sin \varphi + je_2 \cos \varphi$$

when j denotes a rotation of 90° in time. This is well known and can be graphically demonstrated by adding the two sine waves. As shown in Fig. 29, this addition gives a rotating vector of uniform length, the time phase of which will agree with the space angle ϕ.

If these conditions exist, the rotor voltage will remain constant at all angular positions and the dynamometer will give the same reading wherever the rotor of the phase-shifter is turned.

In order to balance the effect of rotor reaction due to the current flowing in the rotor winding, the second phase of the rotor is loaded with the same resistance and inductance as the potentiometer itself, thus producing symmetrical rotor reaction in all positions of the rotor.

To provide the two-phase source necessary for the stator of the phase-shifter it is usual to " phase-split " from a single phase source, as this provides a ready means of adjustment to

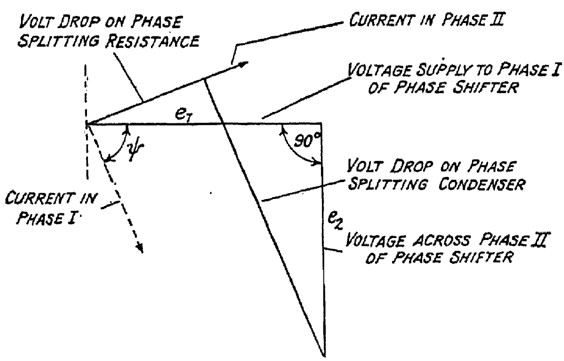


FIG. 30.—Vector diagram of phase-splitting process.

fulfil the requirements of a perfect rotating field in the phase-shifter.

Phase-splitting is brought about by means of a condenser and resistance in series with one phase of the stator winding as shown in Fig. 28.

The vector diagram of the voltage distribution involved in the phase-splitting process is shown in Fig. 30.

The current in phase II must be 90° in advance of the current in phase I, and at the same time the voltage across phase II must be equal to and 90° in advance of that existing across phase I.

The current in phase I lags by the angle ψ behind the voltage

applied to phase I where $\tan \psi$ is the ratio of the reactance to the resistance of this winding. The volt drop on the resistance is in phase with the current in phase II and the volt drop on the condenser lags 90° behind this current.

Because e_1 must be equal to e_2 it will be clear that unless the phase angle of the stator winding is greater than 45° the desired adjustment cannot be made. In practice at 50 cycles the phase angle ψ is about 50° and depends upon the frequency. If the phase-splitting is incorrectly adjusted, the condition for the circular rotating field will not be met; that is, e_1 and e_2 will be unequal and the time phase between them will not be 90°.

In practice the adjustment is made by trial and error, by adjusting the resistance and condenser until the dynamometer remains stationary at all positions of the rotor. A guide to the adjustment is given by the following rule. Adjust capacitance when the phase-shifter axis is at 45° leading. Adjust resistance when phase-shifter axis is at 90° leading.

This process of phase-splitting is perhaps the most difficult feature of this type of potentiometer because it is complicated by fluctuations in the supply voltage which give misleading indications. Fluctuations also occur due to frequency variations and neither of these effects is linear. This disadvantage places a limit upon the accuracy of the indications of the phase-shifter and although precision of magnitude can be maintained by careful readjustment of the current in the potentiometer by means of the rheostats, the phase angles must be determined by triangulation if required accurately.

As will be mentioned later, there are methods of making the phase-splitting a more definite process than in the original instrument, but owing to the limitation of the iron magnetic circuits and the preponderating effect of the air gap the simple rotating field is only approximately achieved in the actual phase-shifter.

Method of Using the Drysdale A.C. Potentiometer.—The latest form of the complete instrument is shown in Fig. 31. This instrument is provided with a changeover switch to facilitate the standardization upon d.c. and the changeover to a.c. without change of connexions.

The process of setting up the instrument may be summarized as follows:

Connect up battery, d.c. galvanometer, a.c. galvanometer, standard cell, a.c. supply, phase-splitting condenser and phase-splitting resistance. Turn the changeover switch to d.c. and balance the standard cell, using the d.c. galvanometer, and adjust the current with the rheostat, having set the potentiometer at 1·01824, *i.e.*, the standard cell value. Set the dynamometer pointer to its 50-milliampere reading by means of the zero adjuster. Turn the changeover switch to the a.c. position and set the phase-shifter axis to zero. Adjust the current in the potentiometer by means of the rheostat until the dynamometer gives the same reading as upon d.c. Turn the phase-shifter to 45° leading and adjust the condenser until the dynamometer pointer reads the same value to which it was adjusted upon d.c. Turn the phase-shifter axis to 90° leading and adjust the resistance until the dynamometer again reads correctly. Repeat this process successively between 0°, 45° and 90° until the dynamometer remains stationary.

A later modification of the circuit has made the standardizing process simpler by the introduction of a mutual inductance in the second winding of the phase-shifting transformer. This enables the adjustment of the phase-splitting unit to be made with greater speed and facility and at the same time extends the utility of the potentiometer by the introduction of some of the advantages of the co-ordinate type of potentiometer which will be described in a later section. Further details of the modifications to the Drysdale potentiometer are given in Chapter X.

When standardized in the above manner the potentiometer is ready for use. Unknown potentials can be measured by applying the unknown to any of the four pairs of test terminals, and turning the selector switch to the appropriate position and balancing the a.c. galvanometer by successive adjustments of the phase-shifter and the dials of the potentiometer.

The process of balancing consists of producing a known voltage exactly similar to the unknown voltage, just as in the case of the d.c. potentiometer, any difference between the two voltages being indicated by the detector or galvanometer. The

phase and the magnitude of the two voltages must agree. The phase of the current through the potentiometer and therefore of the potential drop upon the resistances which compose the potentiometer is rotated by the phase-shifter. The magnitude of the potential is varied by the potentiometer dial setting. The unknown potential is therefore measured as a magnitude with a certain phase relationship. This relationship is purely arbitrary and must be established with reference to some suitable datum. In many cases this datum is provided by the volt drop upon some suitable part of the circuit under test. For example, the volt drop upon a non-reactive resistance will give the phase of the current flowing and this can be made the zero angle so that all other measurements will be with reference to the current in the circuit. Alternatively, the supply voltage may be chosen for the zero phase angle. When the potential which it is desired to use as the phase datum has been balanced, the index of the phase-shifter can be turned to 0° and clamped to the rotor spindle in this position. The "Axis" pointer of the phase-shifter marks the magnetic axis of the rotor so that when this is on zero the rotor volts are in phase with the voltage supplied to phase I of the phase-shifter stator. It is the axis pointer which is used during the process of phase-splitting, but not generally for measurement purposes.

Accuracy.—The accuracy with which potentials can be measured depends upon the accuracy of the Weston Dynamometer Milliammeter and the non-reactivity of the resistance coils of the potentiometer, since the volt drop along any given portion of the potentiometer will be $e = \mathrm{IZ}$ where I is the current and Z the impedance of the potentiometer between the potential points.

The reading of the Weston Dynamometer can be reproduced at the one position on its scale at which it is standardized against the standard cell upon direct current to a precision of rather better than 0·2 per cent. This, therefore, represents the limit of absolute precision with which any a.c. voltage can be determined with the standard form of the potentiometer. This precision will be further reduced by any frequency errors in the dynamometer or effects due to stray magnetic fields, or by the presence of harmonics in the wave-form of the current through

the potentiometer. Although this appears a low degree of accuracy, it is very difficult to obtain a higher degree of absolute accuracy in alternating current measurements by any means, because very few sources of supply are steadier than this. A much higher degree of relative accuracy can be obtained, because fluctuations do not necessarily affect relative measurements.

Wave-Form of Current.—The dynamometer indicates the effective or r.m.s. value of the current. The potentiometer can only balance the fundamental frequency except in the remote case of the harmonics of the wave to be balanced coinciding both in phase and magnitude with those in the potentiometer circuit. This cannot occur in different parts of any reactive circuit even if it should occur in one part, so that in general it may be definitely said that the potentiometer balances only the fundamental frequency and the very selective nature of the tuned vibration galvanometer facilitates this selection.

It is important, therefore, that the wave-form of the current through the phase-shifting transformer should be a good sine wave at all positions of the rotor, otherwise a voltage measurement at one phase angle will not agree with the measurement at another because the harmonics affect the r.m.s. value which the dynamometer indicates. With modern supply systems of good wave-form this condition is very closely fulfilled, so that the current in the potentiometer remains sensibly sinusoidal.

Fluctuations in the Supply Frequency.—The coils in the potentiometer are nearly non-reactive, having a time constant of not more than 10^{-6} seconds and the error resulting from the change in impedance with frequency is negligible compared with other uncertainties. The precision with which phase angles can be measured is dependent upon the phase-splitting. With a perfectly steady frequency and voltage supply the phase-shifter gives electrical angles in agreement with the geometrical angles of the dial to about 0·1 to 0·2 degree. Small phase differences are best measured by vector triangulation, that is by measuring the three voltages which will define the angle. When the frequency of the supply is fluctuating the phase of the rotor current also fluctuates, so that care must be used when interpreting the phase-shifter readings, particularly at large phase displacements. The frequency fluctuations will also

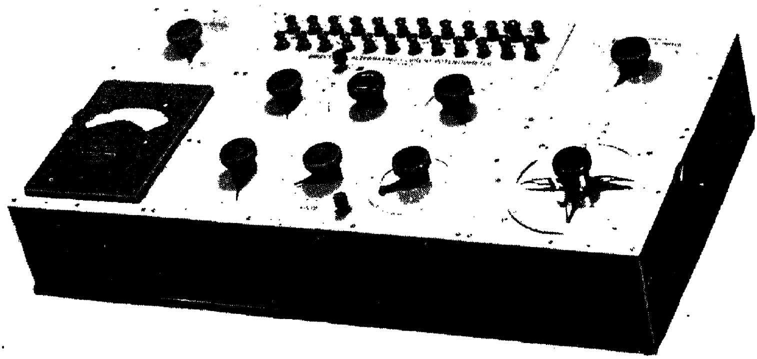


FIG. 31.—The Drysdale a.c. potentiometer.
(H. Tinsley & Co.)

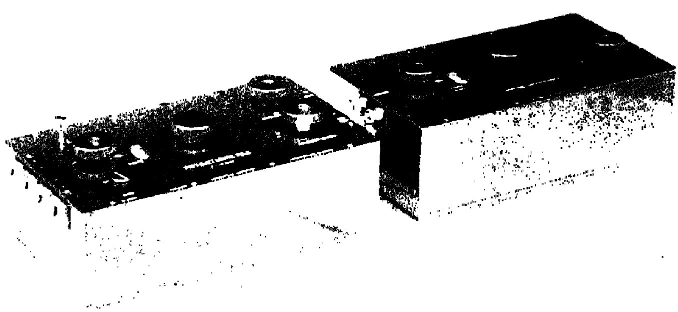


FIG. 33.—Larsen potentiometer.

affect the current flowing through the potentiometer, and it may be necessary to adjust the current by means of the rheostat to make the dynamometer, and therefore the potentiometer, read correctly. It must be borne in mind, however, that if the frequency of the supply is fluctuating there will be a definite restriction placed upon the precision with which the reactive properties of the circuit under test can be determined, because these very properties will be as uncertain as the fluctuations themselves. This can hardly be regarded as a defect in the potentiometer.

Fluctuations in the Supply Voltage.—Fluctuations of the voltage supply may not be so serious, for if the potentiometer and the circuit under test are fed from the same supply, both circuits will vary in step and the balance between them will remain quite steady. This effect is of great importance in a.c. potentiometry because it permits measurements to be made with an accuracy superior to the steadiness of the supply. The measurements so obtained refer to the steady conditions at which the potentiometer reads correctly. In practice, it is difficult with a phase-shifter in the circuit to distinguish between voltage fluctuations and errors of phase-splitting.

Extraneous e.m.f.s.—Care must be taken to avoid another source of errors in a.c. potentiometry. E.m.f.s will be induced in any of the potential leads where there are stray alternating magnetic fields. These e.m.f.s will generally behave as small voltages added vectorially to the voltage under test. Their magnitude can be determined by short-circuiting the leads at the point of measurement and balancing the residual voltage induced in the leads.

To reduce these parasitic e.m.f.s all the potential leads should be twinned so that they do not easily pick up induced voltages. As far as possible, all magnetic apparatus should be kept well away from the potential circuit. Current-carrying leads also should be twinned where this can be done. The phase-shifter in the potentiometer circuit can be the cause of a difficult class of stray induced voltages, which rotate in phase according to the position of the rotor. This means that they cannot be deduced from a measurement, as their values are not constant, but their effect depends upon the actual rotor

position when a measurement is made. Every effort should be made to eliminate this complication when precise work is being done. There is an appreciable capacitance between the stator windings of the phase-shifter and the potentiometer windings, and this allows capacitive current to enter the potentiometer circuits which may disturb the balance. Methods of dealing with this possibility are dealt with in Chapter VIII.

Operation from Three-Phase Source.—For some purposes it is convenient to operate the potentiometer from a three-phase source. For this purpose the phase-shifting transformer is provided with a three-phase winding. When used in this way, frequency fluctuations will not affect the phase relationship of the rotor current to any appreciable extent and this method of supply has some advantages for laid-out routine works tests such as occur in transformer manufacture. Fluctuations in the voltage being self-compensatory, a precision of measurement higher than the steadiness of the supply can be maintained and the results obtained are in effect those of steady supply conditions. By suitable layout of the circuits they can be made to apply to the nominal voltage at which the tests are desired.

The Larsen A.C. Potentiometer.—The Larsen a.c. potentiometer [8–9] is one of the simplest forms of rectangular co-ordinate instruments. It consists of a resistance potentiometer in series with a variable mutual inductance, the whole being connected to an isolating transformer. The circuit is shown schematically in Fig. 32.

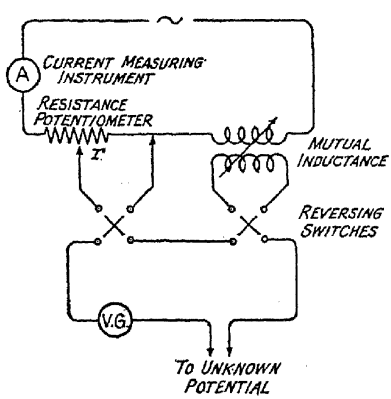


FIG. 32.—Larsen potentiometer-circuit diagram.

The voltage drop Ir on the resistance r of the potentiometer will be in phase with the current I. The voltage induced in the secondary of the mutual inductance will be in quadrature with the current and will be $j\omega$MI where M is the mutual inductance

and $\omega = 2\pi f$, f being the frequency. The unknown potential difference is connected to the secondary of the mutual and the tapping points of the resistance potentiometer through two reversing switches which allow both positive and negative values of inphase and quadrature components to be obtained.

Although the circuit is so extremely simple there are some drawbacks in practice.

The values of M necessary at low frequencies are rather large, or else large currents are necessary. The phase relationship between the isolating transformer (not shown in Fig. 32) secondary voltage and the current it supplies to the resistance potentiometer and primary of the mutual inductance is dependent on the ratio of resistance to inductance, that is, the time-constant of the circuit. Since the time-constant is low, due to the relatively large inductance, the phase angle is relatively large, and therefore any variation of frequency altering the reactance of the circuit will also alter the phase angle. In other words, frequency fluctuation will cause a phase swing of the energizing current. This effect may be reduced by the insertion of ballast resistance, but the result is an increase in the operating power required. Further, the value of the quadrature component is entirely dependent upon the frequency, and no means exist to measure it directly. The presence of the large inductance so near the measurement is a source of stray magnetic fields. The relatively high impedance of the circuit makes it rather insensitive for low-voltage measurements.

Generally, the mutual inductance does not read in volts directly but requires conversion by some multiplier, and the dissimilarity of scales of the two parts of the potentiometer is apt to be confusing. It is to avoid this that the design illustrated in Fig. 33 has been developed. It will be seen that both potentiometer and mutual inductance have the same external appearance and the value of M is so chosen that the dials are direct reading at 50 cycles per second.

The Campbell-Larsen Potentiometer.—The Larsen potentiometer has been modified by A. Campbell [10] so that both components of voltage can be read directly. The circuit of the Campbell-Larsen potentiometer is shown in Fig. 34. The object of this rearrangement of the circuit is to overcome

the difficulty of using the simple Larsen circuit at different frequencies and to facilitate its use at lower frequencies without unduly large mutual inductance. The modified circuit can be set immediately to the values appropriate to the nominal frequency at which measurements are to be made, so that the resistance potentiometer portion and the mutual inductance portion will both read directly in volts when the current is correctly adjusted. It differs from the simple Larsen potentiometer in that only a portion of the total current passes through the resistance potentiometer, which is arranged to form part of a circuit of constant loop resistance R. The current i_1 pass-

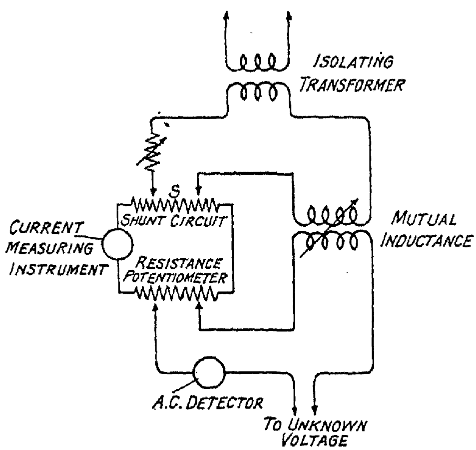


FIG. 34.—Campbell-Larsen potentiometer—circuit diagram.

ing through the resistance potentiometer will always be determined by the ratio of the resistances of the circuit

$$\frac{i_1}{i} = \frac{S}{R}$$

where i is the total current.

If, therefore, the value of S is chosen in proportion to $2\pi f$ the current division will also be proportional to $2\pi f$. Since the voltage induced in the secondary of the mutual inductance is also proportional to $2\pi f$, it follows that by means of this shunt circuit the volt drop in the resistance potentiometer and the induced voltage in the mutual inductance are both proportional to $2\pi f$. By choice of a suitable value of M and S both sections

of the potentiometer become direct reading in volts. In the actual instrument the values of S, controlled by a dial resistance, are calibrated directly in frequency. A thermal device [11] is employed to standardize the a.c. in the circuit. This circuit is shown in Fig. 35. The battery sends a current round the circuit through the resistor K and the heater H when the two double pole switches are in the positions shown. The circuit is so arranged that the volt drop on K is exactly equal to the voltage of the thermo-couple when heated by the current passing through the heater H. The d.c. galvanometer, therefore, reads zero for one current only as any change in the current will disturb the voltage balance and allow the battery to send current through the galvanometer. To standardize the a.c. at the same value the two switches are thrown into the upper position and the heater H is replaced in the battery circuit by an exactly equal resistance H′, thus maintaining the battery current at the same value. If now the a.c. is passed through the heater H and adjusted until the galvanometer reads zero, the r.m.s. value of the a.c. must be equal to the d.c. flowing in the resistance K. With suitably arranged circuit values the d.c. will remain very constant. The constancy can be checked by measuring the volt drop on the resistance K by means of a d.c. potentiometer.

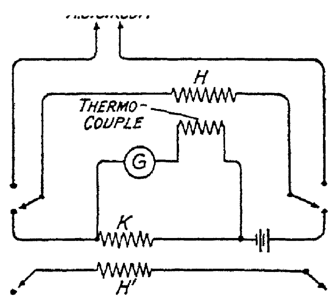


FIG. 35.—Campbell's thermal current balance.

To overcome the need for knowing accurately the frequency in order to set the shunt resistance S to the appropriate value, A. Campbell has made a modification to the circuit which is shown in Fig. 36.[12] The method is to use a differential thermal instrument to obtain equal currents in the two potentiometer circuits. The two heater resistances of the thermal instrument are q in the resistance circuit and p in the secondary of the mutual inductance circuit. When the currents through p and q are equal, the d.c. galvanometer will be balanced. The first step is to set the two switches a and b into the right-hand

position. This closes the circuit of the mutual inductance through the heater p. The impedance of the circuit will, therefore, be made up of the resistance of the secondary circuit, including p and the inductance of the secondary winding of the mutual inductance. That is, the impedance will be

$$r_2 + p + j\omega L_2.$$

The current flowing will be proportional to the induced voltage $e_2 = j\omega M I_1$ and inversely proportional to the impedance.

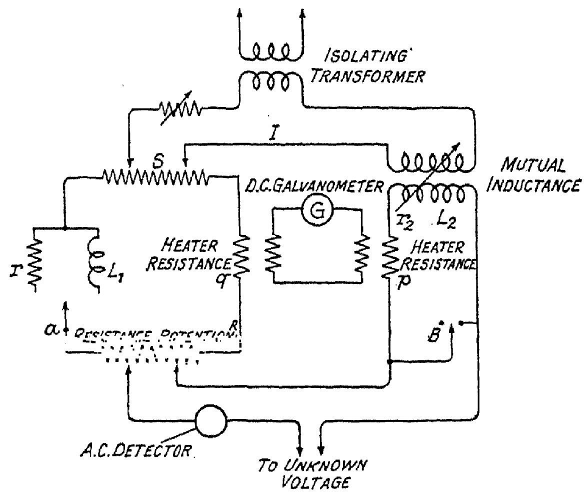


FIG. 36.—Campbell-Larsen potentiometer. Modified circuit.

In the resistance potentiometer circuit the switch introduces the inductance L_1 into the circuit in order to make the time-constant of the two circuits equal. That is

$$\frac{r_2 + p}{L_2} = \frac{R + q}{L_1}.$$

The current flowing in the resistance potentiometer circuit will be proportional to S as previously described. The mutual inductance is set to its maximum value and the value of S adjusted until the d.c. galvanometer shows the two currents to be balanced. Since they are acting in circuits of which the impedances are proportional, the e.m.f.s which produce the current must be in the same proportion. In this way the volt

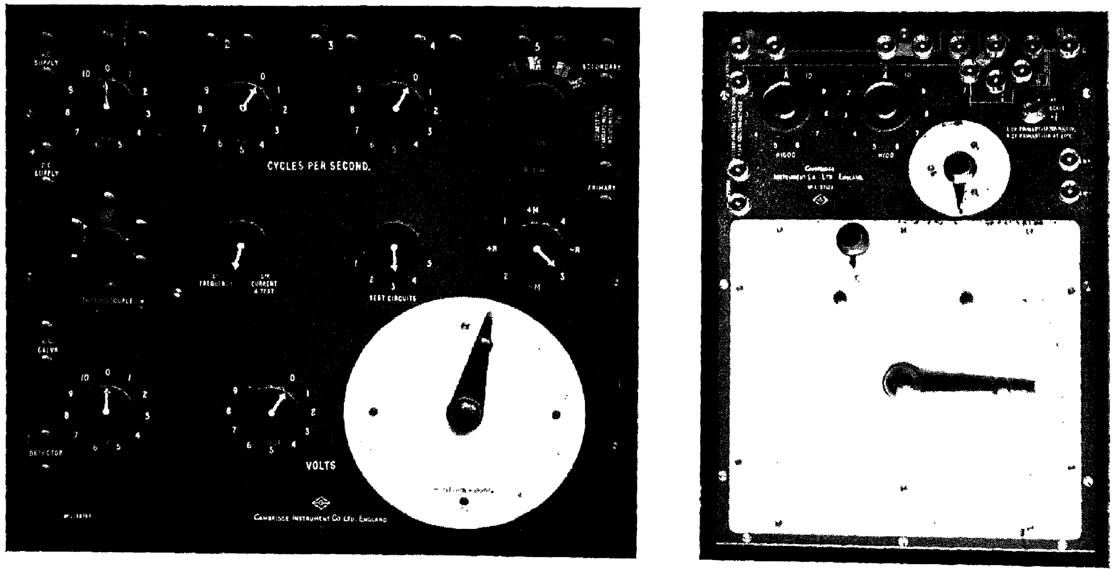


FIG. 37.—Campbell-Larsen potentiometer.
(Cambridge Instrument Coy.)

drop on the resistance S must bear a known relation to the voltage induced in M. As already described, S can be calibrated directly in frequency, and the setting of S so found gives the frequency of the supply.

Having found the correct setting for S in this way the two switches *a* and *b* are moved to the left-hand position. The switch *a* introduces a resistance *r* equal to the resistance of L_1 and so restores the circuit to its non-inductive condition. The switch *b* open-circuits the secondary of the mutual inductance and leaves the potentiometer ready to make measurements. The heater *p* is now switched into a d.c. balancing circuit similar to that shown in Fig. 35 for the standardization of the current.

As will be seen, the potentiometer depends upon the accurate inter-relationship of the circuit values which are somewhat complicated to check. The precision of the thermal current measuring balance is given by Campbell as 1 part in 1,000. The impedance of the potentiometer circuit is rather high owing to the presence of the secondary of the mutual inductance. This makes the measurement of very small potentials difficult, as it restricts the sensitivity of the galvanometer or other detector.

In the instrument made by the Cambridge Instrument Company and shown in Fig. 37 the inphase component has a range of 1·8 volts readable to about 10 microvolts. The mutual inductance gives a range of 1 volt readable to 10 microvolts and to 2 microvolts for small values of quadrature volts if a sufficiently sensitive detector is available. The frequency range covered is from 25 to 1,000 cycles with an accuracy of setting of 1 in 1,000. Higher frequencies involve a simple multiplying factor. The resistance potentiometer can be used also as a d.c. instrument.

The Pedersen A.C. Potentiometer.—The Pedersen a.c. potentiometer [13] makes use of the unique circuit consisting of series resistor R and inductance L in parallel with series resistor R and capacitance C which remains non-reactive at all frequencies provided that $R = \sqrt{L/C}$ and in the two parts of which the currents are always in quadrature.

In the strict sense this is not a potentiometer but a potential

comparator, because in use the actual potentials are not calibrated but used to compare the voltage drop upon known and unknown impedances, so that it is not unlike a bridge in the manner in which it functions. A particular virtue of this circuit is that it can be used for the measurement of frequency and also to check its own quadrature. The simplified circuit is shown in Fig. 38, and the vector diagram in Fig. 39.

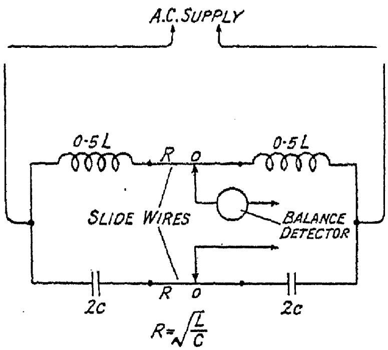


FIG. 38.—Pedersen potentiometer.

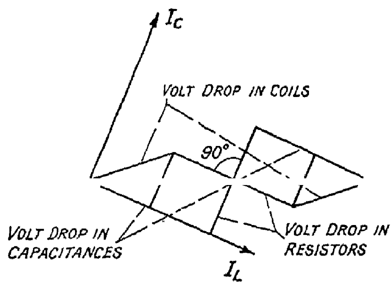


FIG. 39.—Vector diagram of Pedersen potentiometer.

The current in the inductive branch is given by

$$I_L = V/(R + j\omega L)$$

where V is the voltage applied to the potentiometer.

That in the capacitive branch is $I_C = V\Big/\left(R - \dfrac{1}{\omega C}\right)$

and since $CR^2 = L$

these become $\quad I_L = V/R(1 + j\omega CR)$
and $\quad I_C = j\omega CRV/R(1 + j\omega CR)$
whence $\quad I_C = j\omega CRI_L$

which means that the current in the capacitive branch leads that in the inductive branch by exactly 90° at all frequencies. This can be seen from the vector diagram, which also shows that at the centre point of each slide wire the potential is the same, and in consequence both positive and negative values of potential can be selected by the tapping points either side of the centre or zero position on the slide wire. This in effect results in the possibility of measurement of voltage in all four

quadrants without the necessity of reversing the connexions to the tapping points.

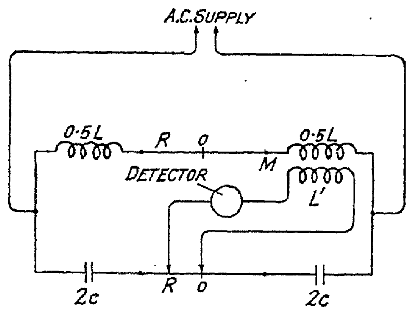


FIG. 40.—Testing quadrature of Pedersen potentiometer.

In using the instrument to measure an impedance it is necessary first to ensure that the currents in the two branches are in exact quadrature and, second, to determine the frequency. The circuit arrangements to fulfil these two conditions are shown in Fig. 40 and Fig. 41. In Fig. 40 the e.m.f. induced in the coil L′ is $j\omega MI_L$ which is leading by 90° on I_L and is thus in phase with the current I_C under correct working conditions. This e.m.f. is balanced by the voltage selected by the tapping points on the capacitive branch by a fraction b of the total resistance of this circuit. Then $j\omega MI_L = bI_C$, or

$$b = j\omega MI_L/I_C = M/CR,$$

that is when M is fixed, the position to be selected by the tapping points on the capacitive branch is also fixed and therefore independent of frequency. The balance condition is checked by the null indication of the detector, and adjustments made by means of the small variable self-inductances shown in the more complete diagram of Fig. 43.

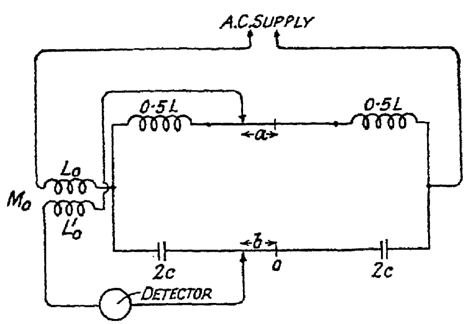


FIG. 41.—Frequency measurement with Pedersen potentiometer.

To measure the frequency, the coil L_0 (Fig. 41) in the main circuit of the potentiometer is coupled to the coil L_0', by the mutual inductance M_0. The e.m.f. induced

in L_0' is then balanced by the voltage drop on the two branch resistances as indicated. The current in the coil L_0 is

$$I_L + I_C = (1 + j\omega CR)I_L,$$

and the e.m.f. induced in the coil L_0' is

$$j\omega M_0(1 + j\omega CR)I_L = (j\omega M_0 - \omega^2 CRM_0)I_L.$$

This is balanced by the settings a and b on the resistances of the inductive and capacitive branches respectively, so that

$$\begin{aligned}(j\omega M_0 - \omega^2 CRM_0)I_L &= aI_L + bI_C \\ &= (a + j\omega CRb)I_L\end{aligned}$$

from which, equating real and imaginary parts

$a = \omega^2 CRM_0$ neglecting the sign for a will simply be negative.

$$b = \frac{M_0}{CR}$$

Hence $$\omega = \frac{}{CR}\ \frac{a}{b}.$$

As in the previous measurement it will be noticed that the setting on the capacitive branch is again independent of the frequency and consequently with a fixed value of M_0, b may be set to some predetermined value and the frequency measured by adjustment of a until balance is obtained.

The measurement of an unknown impedance is effected as indicated in Fig. 42 by comparison of the voltage drop across

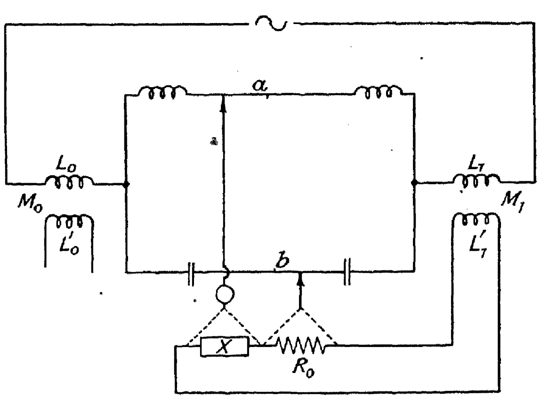


FIG. 42.—Impedance measurement with Pedersen potentiometer.

it with that across a known impedance, usually a standard resistance, carrying the same current. The unknown impedance X and standard resistance R_0 are connected in series to a coil L_1' which is coupled by a variable mutual inductance M_1 to the coil L_1 in the main potentiometer circuit. By variation of M_1 the current in the test circuit can be controlled and the voltage drops across the unknown impedance and standard resistance adjusted to be within the range of measurement of the potentiometer. The voltage across the standard resistance is first balanced by adjustment of the potentiometer tapping points to the values a_1 and b_1 and then that across the unknown

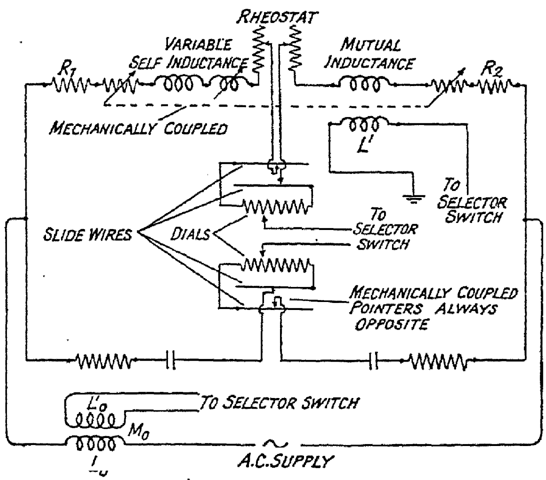


FIG. 43.—Actual circuit of Pedersen potentiometer.

impedance giving the values a_2 and b_2. The voltages thus measured are $(a_1 + j\omega CRb_1)I_LR_S$ and $(a_2 + j\omega CRb_2)I_LX$

from which $$X = \frac{a_1 + j\omega CRb_1}{a_2 + j\omega CRb_2}R_S.$$

Since ωCR and R_S are known, X may be calculated. In certain circumstances it may be possible to adjust the circuit conditions so that $\omega CR = 1$ which results in a simplification of the calculation.

In the instrument as designed for general use a series of dials is used in preference to a slide wire, and it will be seen from Fig. 43 that special arrangements are necessary to give

the potential variations and maintain constant resistance in the circuit.

The Pedersen potentiometer is intended for, and best suited to, impedance measurements at audio and higher frequencies, it is less suitable for power frequencies and measurements of actual voltages or powers. The volt drops upon the two potential systems of which it is formed are usually very unequal and, furthermore, they change phase appreciably with frequency fluctuation. Fig. 44 shows a complete potentiometer of this type in self-contained form for use up to 100,000 cycles per second.

NOTES AND BIBLIOGRAPHY.

[1] Franke, Adolf: "Die elektrischen Vorgänge in Fernsprechleitungen und Apparaten," *Electrotech. Z.*, 1891, xii, 447–52, 458–63.

[2] Ebeling, A.: "Franke's Alternator for measuring Currents of Telephonic Frequency," *Electrician*, 1913, lxxii, 88–91.

[3] Drysdale, Charles V.: "The Use of Potentiometers on Alternate Current Circuits," *Phil. Mag.*, 1909, (6), xvii, 402–11.

[4] Drysdale, Charles V.: "The Use of the Alternate Current Potentiometer for Measurements on Telegraph and Telephone Circuits," *Electrician*, 1913, lxxi, 687–90.

[5] Drysdale, Charles V.: "The Alternate-current Potentiometer as a Standard Instrument," *Electrician*, 1915, lxxv, 157–60.

[6] Drysdale, Charles V.: "Alternating-Current Potentiometers and their Applications," *J. Instn. Elect. Engrs.*, 1930, lxviii, 339–60.

[7] Drysdale, Charles V.: "The Use of a Phase-shifting Transformer for Wattmeter and Supply Testing," *Electrician*, 1908, lxii, 341–3.

[8] Larsen, Absalon: "Der Komplexe Kompensator, ein Apparat zur Messung von Wechselstromen durch Kompensation," *Electrotech. Z.*, 1910, xxxi, 1039–41.

[9] (English Abstract of 8.) "An Apparatus for the measurement of Alternate Currents by the Compensation Method," *Electrician*, 1911, lxvi, 736.

[10] Campbell, Albert: "A New Alternating Current Potentiometer of Larsen Type," *Proc. Phys. Soc.*, 1928, xli, 94–9.

[11] Campbell, Albert: "Alternate-current Measurements," *J. Instn. Elect. Engrs.*, 1901, xxx, 889–908.

[12] Campbell, Albert, and Childs, Ernest C.: "The Measurement of Inductance, Capacitance and Frequency," p. 459, 1935 (Macmillan & Co., London).

[13] Pedersen, P. O.: "A New Alternate-current Potentiometer for Measurements on Telephone Circuits," *Electrician*, 1919, lxxxiii, 523–5.

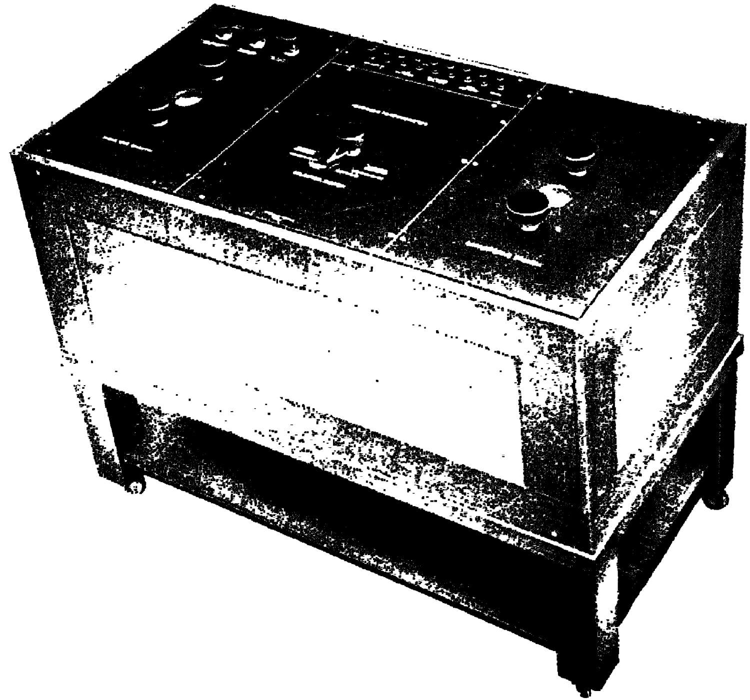


Fig. 44.—The Pedersen potentiometer.

THE A.C. POTENTIOMETER (*continued*).

The Co-ordinate A.C. Potentiometer.—A considerable experience with the polar potentiometer led the author to devise the Gall type of co-ordinate potentiometer [1–5] as a means of avoiding the use of the phase-shifting transformer for phase rotation of the voltage in making a balance. More mature experience has shown that a combination of the two types has many other advantages.

The co-ordinate a.c. potentiometer consists of two exactly similar potentiometers. One is supplied with current substantially in phase with the supply, and the other with current exactly in quadrature with the former. The former is called the "in-phase" potentiometer and the latter the "quadrature" potentiometer.

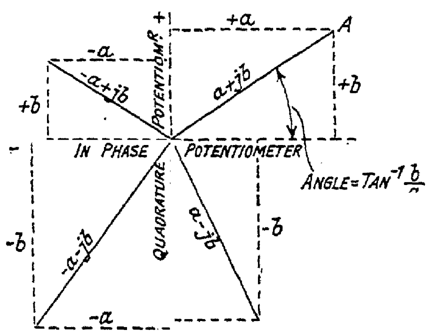


FIG. 45.—Voltage vectors as built up by co-ordinate a.c. potentiometer.

Unknown potentials are balanced by an equal known voltage compounded of two components at right angles derived from the two potentiometers. Each potentiometer has a reversing switch in its potential circuit so that the phase of the component can be turned through 180°, or from positive to negative. The way in which any vector value of voltage can be built up follows from Fig. 45.

The vector voltage OA is made up of the component a upon the inphase potentiometer plus the voltage b upon the quadrature potentiometer giving the resultant value $E = a + jb$ volts

in the usual symbolic notation. The voltage OB will be made up of $-a_1 + jb$, that is, the potential derived from the inphase potentiometer is reversed.

The inphase potentiometer consists of a non-reactive potentiometer having 18 studs on the main dial and a slide wire subdividing the volt drop between the studs. Normally the voltage range is 1·9 volts subdivided to 1 millivolt and readable to 0·1 millivolt with a lower range of from 0·19 volt readable to 10 microvolts. The inphase potentiometer has a sensitive reflecting dynamometer of the torsion head type in series with it. This instrument gives a full-scale deflexion of the torsion head

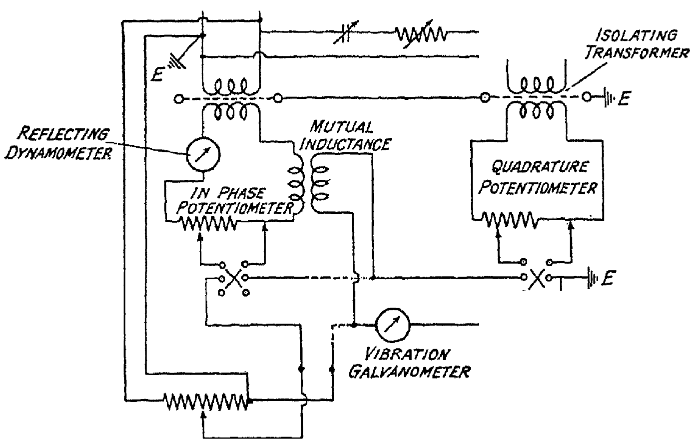


FIG. 46.—Simplified circuit of co-ordinate a.c. potentiometer.

for 50 milliamperes, and this current can be observed upon the optically reflected scale to about 1 or 2 parts in 10,000. The dynamometer reads effective or r.m.s. current without frequency errors up to 2,000 cycles and provides the link between the d.c. standardization against the standard cell and the a.c. measurements to a precision comparable with that of the standard cell itself. The simplified circuit is shown in Fig. 46.

The inphase potentiometer is balanced against the standard cell upon direct current and the torsion head of the reflecting dynamometer turned until the reflected "spot" is brought back to zero. The potentiometer circuit is now fed with a.c. and the current adjusted until the dynamometer spot is again

balanced at zero, the torsion head remaining untouched. This means that the same effective or r.m.s. value of current is flowing through the circuit and the potentiometer is, therefore, giving true values of voltage upon its dials.

In order to standardize the quadrature potentiometer, it is essential that the phase of the current should be adjusted to be exactly 90° displacement from that of the inphase potentiometer as well as at its correct magnitude.

This is done in the following way (see Fig. 46). A standard mutual inductance is included in the circuit of the inphase potentiometer, and the volt drop on the quadrature potentiometer balanced against the secondary voltage induced by the inphase current, by adjustment of the phase-splitting circuit in the primary of the isolating transformer. The secondary winding is disconnected after standardization.

The voltage induced in the secondary of the mutual inductance will be in quadrature with the primary current so that it can only be balanced when the voltage of the quadrature potentiometer also is in quadrature—which is what is required.

Further, by choosing a suitable value of mutual inductance, in this case 31·83 millihenries, the voltage will be some definite proportion of the frequency, because there will be 50 milliamperes in the primary circuit. The induced voltage $e = 2\pi f \mathrm{M}i$, therefore if $\mathrm{M} = \dfrac{1}{10\pi}$ henries $e = \dfrac{f}{100}$ volts. If, therefore, the dials of the quadrature potentiometer are set to 0·5000 volt at 50 cycles and the supply to the quadrature potentiometer adjusted until secondary voltage of the mutual inductance is balanced, the quadrature potentiometer will be standardized against the inphase potentiometer both for phase and magnitude. The accuracy of the magnitude will depend upon the accuracy with which the frequency is known. If this is a source of doubt then a second dynamometer should be included in the quadrature potentiometer to standardize the current to 50 milliamperes. The mutual inductance gives the quadrature accurately, independently of frequency, while the setting of the balanced potentiometer will then give the frequency of the supply. It is seldom found necessary to employ this second dynamometer because the frequency of the supply is usually known.

The potentiometer is usually provided with a changeover board containing the necessary switches for changing over to d.c. for standardization. A separate rheostat is included in the d.c. circuit so that it is not necessary to disturb the a.c. setting of the circuit. This allows the standard cell balance to be checked quickly without any readjustment on the a.c. side. To supply the quadrature potentiometer some form of phase-splitting circuit is employed. It is desirable that there should be no appreciable harmonics in either of the circuits, since the potentiometer gives essentially a single frequency balance. Originally a form of mutual inductance, phase-splitting was employed in preference to condenser phase-splitting with the

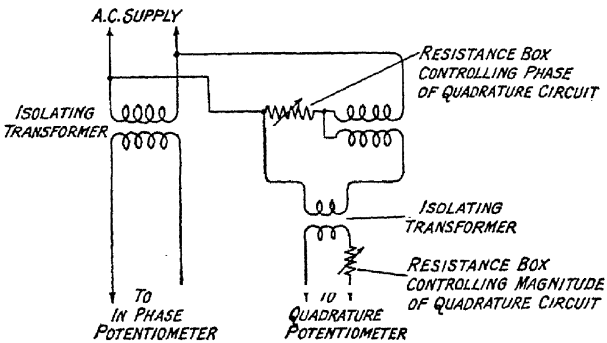


Fig. 47.—Original phase-splitting circuit for co-ordinate a.c. potentiometer.

object of reducing the magnification of harmonics by the capacitance (Fig. 47). With the advent of nickel-iron magnetic alloys it became possible to change to condenser phase-splitting. Much more efficient phase-splitting circuits can be constructed, reducing the phase-splitting losses to negligible dimensions, so that the whole potentiometer can be operated with less than 1 watt, instead of about 40 watts with mutual inductance phase-splitting. This makes it possible to use the potentiometer upon a valve oscillator of reasonable output.

The isolating transformers are essential to prevent cross-connexion between the potentiometer and the circuits under test. They also serve the useful purpose of reducing the capacitance between the supply and the potentiometer. The

importance of this will be dealt with later. The latest method of phase-splitting is shown in Fig. 48.

Both isolating transformers are provided with screens to their primary and secondary windings, as well as between the windings, so that capacitance current from the source can be prevented from entering into the potentiometer circuit.

The two isolating transformers are usually built into one unit and can be placed remote from the instrument; a point which helps to reduce sources of stray fields. The resistance and condenser are built in one unit of convenient size for placing behind the potentiometer for easy control. The standard mutual inductance and the dynamometer are both astatically

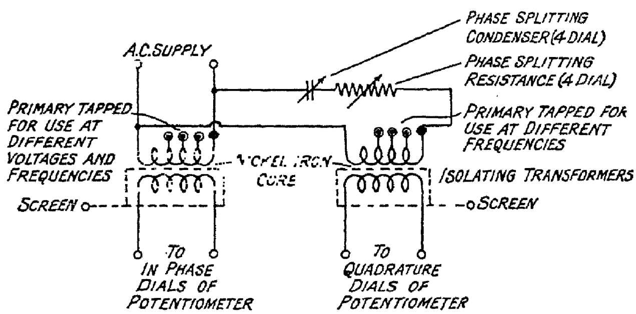


FIG. 48.—Phase-splitting circuit for co-ordinate a.c. potentiometer.

wound but should be placed where they are unlikely to cause or be affected by, stray magnetic fields.

Owing to the inductance of the dynamometer and the primary of the mutual inductance in the inphase potentiometer circuit, the current in the circuit will be displaced in phase with respect to the supply voltage. This means that the inphase potentiometer is not truly in phase with the supply but lags behind it. For many purposes this may not be important, but for some purposes it is an advantage to be in phase with the supply volts. When this is required, it is necessary to put the dynamometer and mutual inductance in the quadrature potentiometer and leave the inphase potentiometer entirely non-reactive. It makes no difference to the operation of the instrument but is rather more extravagant in the required

power, because this is largely governed by the phase-splitting circuit.

It can be shown [6] that the minimum possible conditions of phase-splitting will require reactances storing energy equal to at least twice the voltamperes of the quadrature circuit, thus the larger the voltamperes of this circuit the more extravagant the phase-splitting.

The dynamometer has an inductance of 50 millihenries and a resistance of 87 ohms. The primary of the mutual inductance has a resistance of 20 ohms and an inductance of 30 millihenries. The resistance of each potentiometer is 40 ohms.

The complete circuit of the potentiometer, including the changeover board, is shown in Fig. 49.

It will be noticed that there are two "test" positions. These are arranged so that the voltage of either potentiometer can be used independently. For example, in standardizing against the standard cell only the inphase potentiometer is used, that is, the selector switch is turned to Test 1 position. The selector switch is turned to Test 2 position when balancing the quadrature potentiometer against the mutual inductance. Four other positions of the selector switch are available for testing purposes. Each potentiometer is fully screened electrostatically and all the connexions between the selector switch and the terminals are screened. This permits the use of the instrument at audio and "carrier" frequencies if suitable precautions are taken to screen the circuits effectively.

The sign-changing switches reverse the potential points of each potentiometer. The potential drop on the two potentiometers is added by being connected in series so that the voltage is made up of the volt drop a on the inphase potentiometer, plus the volt drop b on the quadrature potentiometer, and can have any value

$+a + jb$ both signs $+$
$-a - jb$ " " $-$
$+a - jb$ inphase sign $+$ quadrature sign $-$
$-a - jb$ " " $-$ " " $+$

In this way the resultant potentiometer voltage can be turned into any quadrant of the circle.

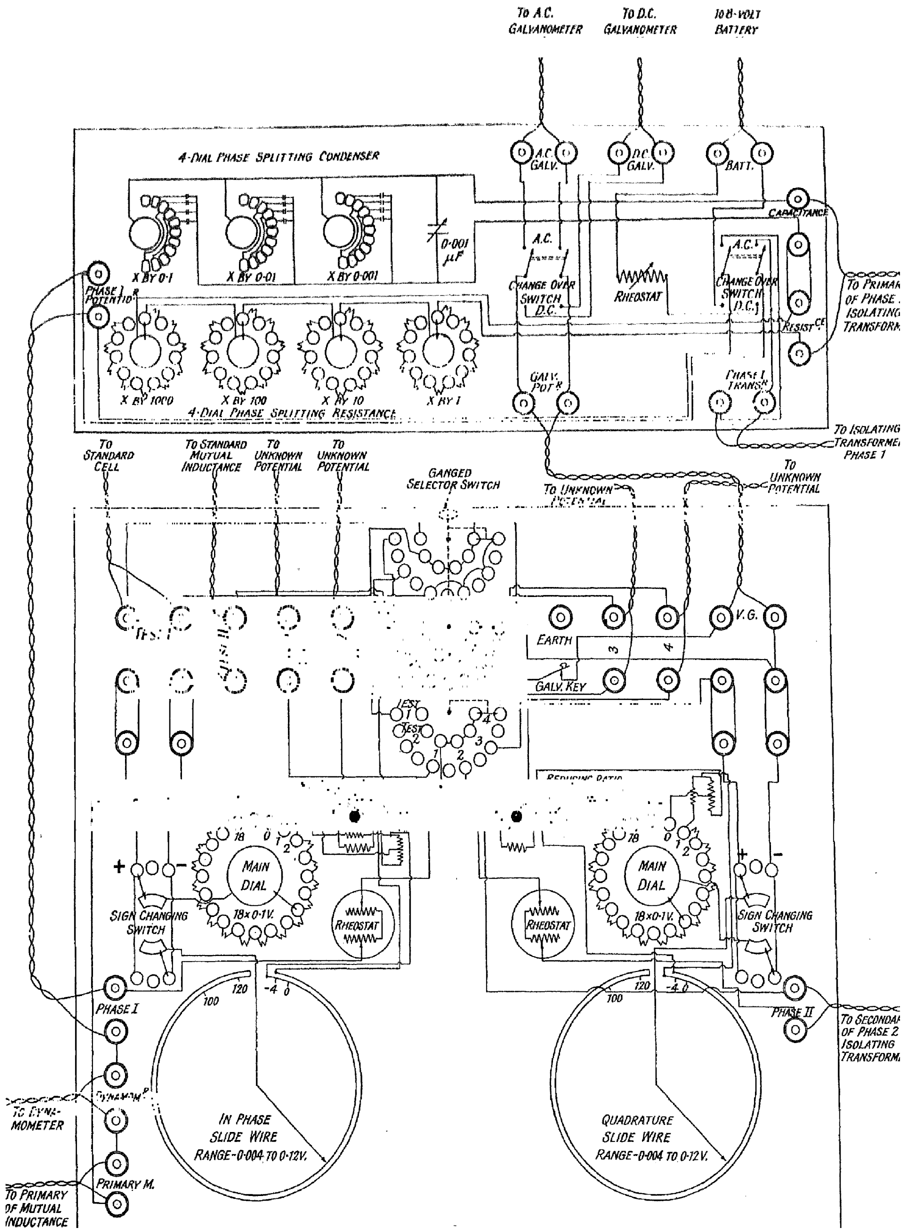


Fig. 49.—[illegible]

[To face page 83

The co-ordinate potentiometer can be used also as an acc ate d.c. potentiometer. The degree of subdivision can increased if desired by using the two potentiometers of whi it is composed in series, supplied from separate batteries from the same battery using the reduced range for one potent meter. This combination gives a very highly subdivided instr ment. A special advantage of the two separate potentiomete is that one can be used to calibrate the other by the meth described in Chapter XII.

General Procedure in using the "Co-ordinate" A. Potentiometer.—For work at low frequencies, such as cycles, the potentiometer equipment consists of the followi items: The potentiometer itself, the isolating transformers, t phase-splitting unit and changeover board, the reflectir dynamometer, the standard mutual inductance, vibration ga vanometer shunt and the standard cell.

The apparatus is arranged as shown in Fig. 50. The sel contained reflecting galvanometers shown there are the mo suitable type because they can be set up in any position witl out separate scales, allowing close grouping of the scales, whic is very convenient in operation. The reflecting dynamomete shown in Fig. 51 upon which the standardization of the instr ment depends should be set up so that the scale can be easil observed and the torsion head easily reached for adjustmen The dynamometer is mounted with its own optical system i a box with a cover which can be closed when not in use.

The two galvanometers and the reflecting dynamomete can most conveniently be mounted immediately above an behind the potentiometer with the scale slightly below eye level. It is much less tiring to look down, rather than up, t the scales.

The isolating transformer and the standard mutual induc tance should be as far away as convenient from the potentio meter and from each other. The phase-splitting unit an changeover board should be just behind the potentiometer, i easy reach for adjustment when required.

The isolating transformer is usually arranged for 100 to 11 volt supply. It is convenient to have a voltmeter across th supply to the potentiometer. The main power terminals t

which the apparatus under test is to be connected should also be on this circuit. It is preferable to have the control rheostat on the supply side of these terminals so that the voltage to the potentiometer is always the same as that to the test circuit, but that may not be possible if the currents to be dealt with are too large.

The changeover board is provided with switches so that the 8-volt battery can be connected to the inphase potentiometer for the purpose of standardizing the dynamometer against the standard cell. This is done by setting the dials of the inphase potentiometer to the value of the standard cell, 1·0183 volts at 18° C. and adjusting its rheostat until the d.c. galvanometer shows that the potentiometer balances the standard cell. The switches of the changeover board change over the supply from d.c. to a.c., connecting the a.c. galvanometer in place of the d.c. galvanometer and the standard mutual inductance in place of the standard cell. The selector switch in the potentiometer connects the potentiometer to the appropriate test terminals.

When the standard cell has been balanced in this way, the torsion head of the reflecting dynamometer must be turned until the "spot" is brought back to its original zero position. A tangent screw is provided to give a very fine control of the torsion head, and the sensitivity is such that current changes of 1 part in 10^4 will be detected by the "spot" of the reflecting dynamometer. The current when standardized in this way is 50 milliamperes and this corresponds to one complete turn of the torsion head. The dynamometer should be re-standardized from time to time during tests because the spring weakens with temperature rise due to the self-heating of the dynamometer coil. Constancy is reached in about half an hour on full current.

The changeover board is now switched over to a.c. and the selector switch in the potentiometer turned so that the quadrature potentiometer is connected to the secondary of the mutual inductance. The current through the inphase potentiometer must first be adjusted to reproduce the same deflexion upon the reflecting dynamometer as when standardized upon d.c. When this adjustment has been made by means of the rheostat in the potentiometer, the same r.m.s. value of current will be passing through the inphase potentiometer. The spot of the

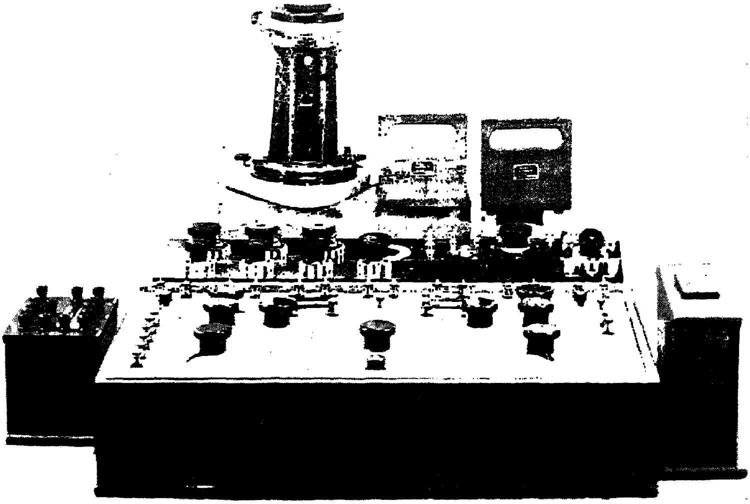
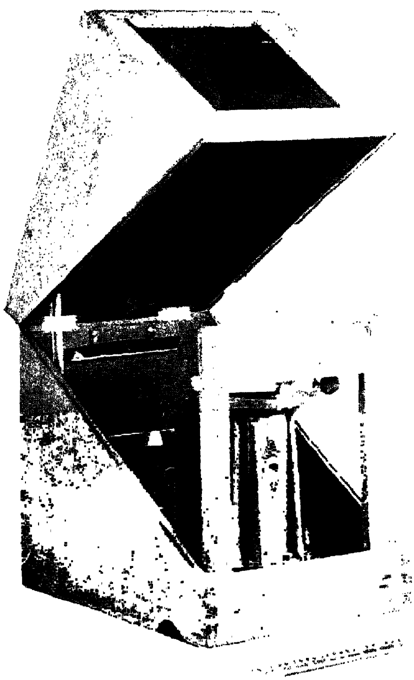


FIG. 50.—Arrangement of the complete co-ordinate a.c. potentiometer on th
test bench. (H. Tinsley & Co.)



dynamometer usually exhibits a certain amount of unsteadiness upon a.c. due to the fluctuation in the supply. The magnitude of these is an indication of the limit of precision to which the absolute value of potential can be measured, but not necessarily the limit of precision in carrying out a test, because of the self-compensating action of the circuit when balanced against another voltage varying in exactly the same way.

Having calibrated the inphase potentiometer, the quadrature potentiometer is then standardized against the mutual inductance. The dials of the quadrature potentiometer are set at the known frequency, for example at 50 cycles to 0·500 volt, and the phase-splitting adjusted until the vibration galvanometer is balanced, showing that there is no difference of potential between the quadrature potentiometer and the secondary of the mutual inductance. The current through the inphase potentiometer passes through the primary of the mutual inductance, so that the secondary voltage is in quadrature with this current, and therefore in quadrature with the volt drop upon the non-inductive coils of the inphase potentiometer winding. By balancing the quadrature potentiometer against this secondary voltage, the two potentiometer voltages are brought into quadrature with each other. The magnitude of the secondary voltage will be $E = \omega M \times 50 \times 10^{-3}$ volts because the current in the inphase potentiometer is 50 milliamperes.

If M is $31{\cdot}83 \times 10^{-3}$ henries

$$\begin{aligned} E &= 6{\cdot}284 \times 31{\cdot}83 \times 10^{-3} \times 50 \times 10^{-3} f \\ &= f/100 \text{ volts.} \end{aligned}$$

It is essential that the frequency be known with sufficient accuracy, if the quadrature potentiometer is to be standardized in this way. If the actual frequency of the supply differs slightly from the specified frequency, the e.m.f. induced in the standardizing mutual inductance and applied to the quadrature potentiometer tapping points will be $j2\pi f_1 MI$ instead of $j2\pi f MI$. Since the tapping points are set at the voltage corresponding with f the resulting quadrature energizing current, whilst still being in quadrature with the inphase current, differs from its correct value by an amount equal to the ratio of the actual to the true value of the frequencies. A correction is therefore

necessary; all values obtained on the quadrature potentiometer must be multiplied by f_1/f. This effect may be of importance in certain types of measurement where high accuracy is required and will be discussed in greater detail later.

The adjustment of the phase-splitting circuit is made by successive approximations until a sharp balance of the vibration galvanometer spot is obtained. In the preliminary stages, the galvanometer shunt should be used to reduce the sensitivity of the galvanometer. If the galvanometer spot shows a number of images and cannot be brought to a sharp balance, it is due to the presence of harmonics in the supply. The standardizing balance is a very severe test for the presence of these because the induced voltage in the secondary of the mutual inductance is proportional to the frequency and the harmonics are, therefore, magnified up in proportion to their order.

The presence of some small harmonics in this balance, with the galvanometer completely unshunted, is almost inevitable, but need not seriously affect the use of the instrument. They must reduce the accuracy because the current is standardized by its r.m.s. value, which includes the harmonics, whereas the actual potential balances are made on the fundamental only.

The error is, therefore, the difference between the r.m.s. value of the fundamental and the fundamental plus harmonics. It has been pointed out by Dr. C. V. Drysdale [7] that a single 14 per cent. harmonic or two 10 per cent. harmonics are necessary to produce a 1 per cent. error. A single harmonic of 4·5 per cent. would produce an error of only 0·1 per cent. and each 1·4 per cent. harmonic produces an error of 0·01 per cent.

In practice the blurring of the vibration galvanometer spot in the unshunted condition on the mutual inductance balance should not exceed about 0·5 cm. on the scale for good supply condition, and reasonably good work can be done when the blurring amounts to several centimetres. Shunting the vibration galvanometer will reduce the apparent effect of the harmonics.

The Ayrton-Mather type of universal shunt, in which the resistance in the galvanometer circuit remains constant, is not suitable for use with the vibration galvanometer. It is better to employ a special type of galvanometer shunt which will

greatly damp the galvanometer when reduced sensitivity is required. When the vibration galvanometer is heavily damped by a low resistance across the vibrating coil, its response is very rapid and it will follow changes in the balance as quickly as they can be made. This facilitates quick balancing, and the shunt can be removed to provide the maximum sensitivity to perfect the balance finally. It is advisable to introduce resistance in series with the shunted galvanometer to prevent excess current flowing round the galvanometer circuit when the potentiometer is much out of balance. This resistance can be incorporated in vibration galvanometer shunts and can be cut out, in the position of maximum sensitivity, when the galvanometer is unshunted. Unless this series resistance is introduced when small voltages are being measured, the value of which is quite unknown, difficulty is sometimes experienced in deciding in which direction to adjust the potentiometer dials. A false minimum deflexion of the vibration galvanometer spot may be obtained if the impedance of the galvanometer circuit is too low. When this occurs any movement of the dials increases the deflexion, but no balance can be obtained. It is due to resistance of the slide wires and coils in the potentiometer circuit being higher than the galvanometer circuit across them, and can only occur when the galvanometer is heavily shunted. Although out of balance, the actual current flowing round the circuit decreases as the potential points on the slide wires and coils are separated, because the resistance of the circuit in series with the galvanometer is increasing faster than the volt drop on the shunted wires.

The actual procedure in balancing is to make successive adjustments upon the two potentiometers and observe the vibration galvanometer. If the deflexion decreases then the adjustment should be continued in that direction until the minimum deflexion is obtained. The other potentiometer should then be adjusted in the same way. If the potentiometer being adjusted reaches zero in the process, the sign-changing switch should be operated and the setting then increased. This reversal of sign takes this component of voltage through zero. It is a convenient preliminary to making a balance to operate both the inphase and quadrature sign-

changing switches until the minimum position is found. This usually ensures that the phase of the two components is in the right direction. The effect of the sign-changing switch is simply to reverse the potential points. This is equivalent to changing the phase through 180° or reversing the sign. The sign of any voltage being measured can be changed in this way by simply reversing the leads to the potentiometer terminals.

When an approximate balance has been obtained the galvanometer shunt can be opened and the balance perfected. The key can be left depressed during the final stages as there will be no danger of the small out-of-balance current damaging the potentiometer or galvanometer, but it should always be open to protect the circuit during other changes.

When the final balance is obtained, the results are read directly on the two potentiometers as a complex quantity.

It is most important to note the signs. If the test circuit leads are reversed the setting of both potentiometers should remain unchanged, the signs only being reversed. Unless reversing the leads gives exactly the same reading reversed, there are induced e.m.f.s acting in the circuit.

In many laboratories, where highly inductive apparatus is tested, large magnetic fields are present. These may cause false readings by affecting the potential circuits. That is, those circuits in which no current flows when a balance has been obtained. Induced voltages will appear in these circuits and give a false condition of balance. The secondary of the mutual inductance is astatically wound to reduce this effect, but a very unsymmetrical stray field would cause a considerable voltage to be induced into this winding. Usually it can be eliminated by orientating the inductance until no voltage is induced, using the vibration galvanometer across the terminals to find the zero position of induction. All the leads in the potential circuits and the galvanometer leads should be carefully twinned to eliminate induced voltages. The galvanometer shunt must be non-inductive, otherwise it will act in the presence of stray fields as an inducing coil straight across the galvanometer terminals. The simple test is to short-circuit the ends of the potential leads between which the potential is to be measured and then balance the galvanometer. The potentiometer dials

should then read zero. It is very seldom that there is any difficulty in bringing about this condition. Occasionally, tests have to be made with the potential leads widely separated, due to the formation of the apparatus under test. The voltage induced by the stray field in these cases can be allowed for by a separate measurement of the e.m.f. induced in the leads. In the case of measurements at higher frequencies, it is necessary to take particular care to eliminate stray inductance and capacitive effects, and if suitable precautions are adopted, the a.c. potentiometer can be used up to 100 kilocycles. It is advisable to rearrange the circuit for these higher frequencies and omit the dynamometer, but at audio frequencies up to, say, 2 kilocycles, the low-frequency arrangement of the circuit is suitable. The inductance of the dynamometer necessitates a rather higher voltage supply to that circuit. A smaller value of mutual inductance should be used at audio than at power frequencies.

In the high-frequency arrangement of the circuit, shown in Fig. 52, the dynamometer is removed, and small toroidally wound high-frequency mutual inductances, M_1 and M_2, are inserted into both the inphase and the quadrature potentiometer circuits, to enable the correct phase and equality of the respective currents to be checked. A special isolating transformer is required. The standardization of the potentiometer is made upon a potential divider across the supply with a valve voltmeter to measure the supply voltage. The accuracy is necessarily limited to the means available for measuring high-frequency voltage.

It is generally advisable to work with one side of the test circuit at "earth" potential, and to arrange the circuits so that they are screened from stray capacitive effects. The isolating transformer for supplying the potentiometer is provided with screens to each winding. By connecting these screens to one side of the supply, all leakage and capacitive currents will be by-passed back to the supply. The screen between the windings can be earthed so that capacitive currents will flow directly to "earth" instead of into the potentiometer circuit. One end of the potentiometer circuit can be connected to its own screen so that no part of the measuring circuit is more than

2 volts above earth potential. The circuit under test must be so arranged that one of the points between which the voltage is to be measured is at earth potential as shown in Fig. 52. The potential leads should be screened, but it must be remembered that the capacitance of these leads will be shunted across the circuit under test and across the potentiometer circuit and the effect of this capacitance may be of importance. It may be necessary to provide fixed potential leads to the apparatus to be tested and to bring these leads close to the potentiometer, so that their effect can remain constant and be allowed for.

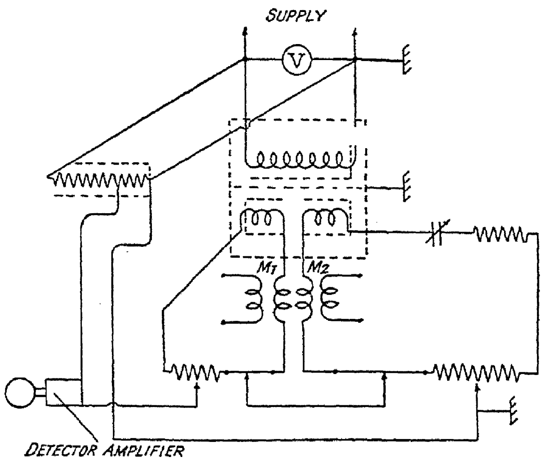


FIG. 52.—Co-ordinate potentiometer arranged for use at the higher frequencies.

This precaution applies to measurements made upon circuits of high impedance.

When it is necessary to obtain a greater degree of sensitivity a valve amplifier may be used in the galvanometer circuit. For high-frequency measurements the detector usually consists of a heterodyne detector amplifier. These will be dealt with in a later section.

A Portable A.C. Potentiometer.—A portable a.c. potentiometer[4] was designed by the author for workshop and field measurements. This potentiometer is self-contained with a sensitive vibration galvanometer built in to provide the detector. The illumination of the galvanometer spot is provided from

the a.c. supply from which the potentiometer is excited. This is usually from 100 to 110 volts. A number of fixed frequency ranges can be provided and these have covered from 25 cycles per second up to 1,000 cycles per second. The potentiometer consists of two dials, one giving inphase potentials and the other giving quadrature potentials. The range usually constructed covers from 0 to $\pm$ 150 millivolts on each slide wire. A volt ratio box is incorporated which extends the range to 1·5 volts, 15 volts and 150 volts, and a set of resistances for the measurement of currents up to 15 amperes. The potentiometer can also be provided with reducing ratios so that voltages down to 1 microvolt correspond to the least count. This instrument has found a useful application in routine iron testing, general a.c. relay design, and in the measurement of magnetic fields in connexion with geophysical prospecting for minerals. Fig. 53 shows the simplified circuit diagram. The circuit is very simple in theory, but considerable care is necessary in practice to avoid electro-magnetic pick-up into the galvanometer from the components all housed in the same case. The supply is connected through a controlling rheostat to an isolating transformer. This transformer is provided with three secondary windings. One is used to illuminate the vibration galvanometer

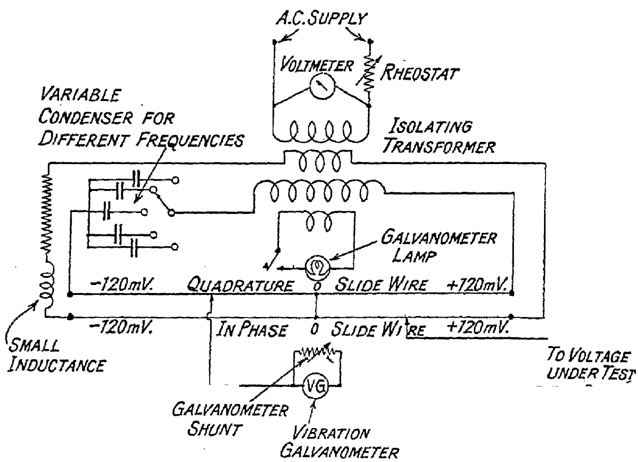


FIG. 53.—The Gall portable a.c. potentiometer circuit.

lamp. The second winding supplies current to the quadrature wire through a condenser in series with the slide wire. The third winding supplies the inphase wire. The time constant of this circuit is adjusted by a small inductance to make the volt drop on the inphase wire in true quadrature with the quadrature wire.

The two centre or zero points of the slide wires are connected and each wire reads positive and negative values on each side of the zero point. Various fixed frequency ranges are obtained by altering, by means of a ganged switch, the condenser and inductance values. A voltmeter across the supply is used for standardization, calibration points for each frequency being marked upon the scale. The pointer is adjusted to these calibration points by means of the controlling rheostat which allows of a wide variation in the supply voltage.

A further type of portable instrument for measuring the phase and magnitude of the voltage has been devised by the author. It is shown in Fig. 54. This is called the vector voltmeter.[5] It consists of a pointer type of instrument which measures the two components of an a.c. voltage just like a co-ordinate potentiometer. The meter resistance is made as high as possible so that it does not appreciably disturb the circuit to which it is connected. The instrument has two windings called, for convenience, the local winding and the control winding respectively. The local winding is connected to the mains at any convenient voltage such as 100 or 230 volts. The control winding, which is of very high resistance, is connected to the circuit of which the voltage has to be measured.

The instrument is provided with a switch for standardizing, an inphase or quadrature switch, and a sensitivity control rheostat. The first switch in the test position changes the connexions of the two windings so that the fixed and moving coils are in parallel across the supply or local voltage. With the second switch in the inphase position the instrument then behaves as a dynamometer voltmeter and reads the true voltage of the local circuit provided the control rheostat is turned to the open circuit position. (It should be noted that both circuits contain large series resistances and their time-constants are equal.)

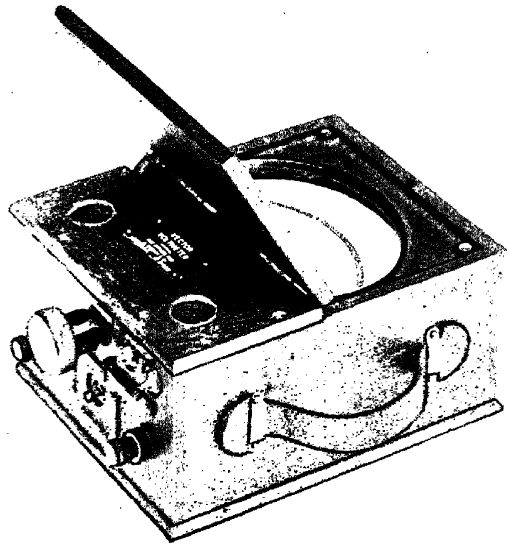


FIG. 54.—The vector voltmeter.

Having measured the voltage of the supply in this way upon the true voltage scale, the sensitivity control rheostat is then turned until the pointer gives the same reading upon the Vector voltage scale. This means that the vector voltage scale has been standardized to suit the true value of the local supply voltage. Any voltage can then be measured by the control

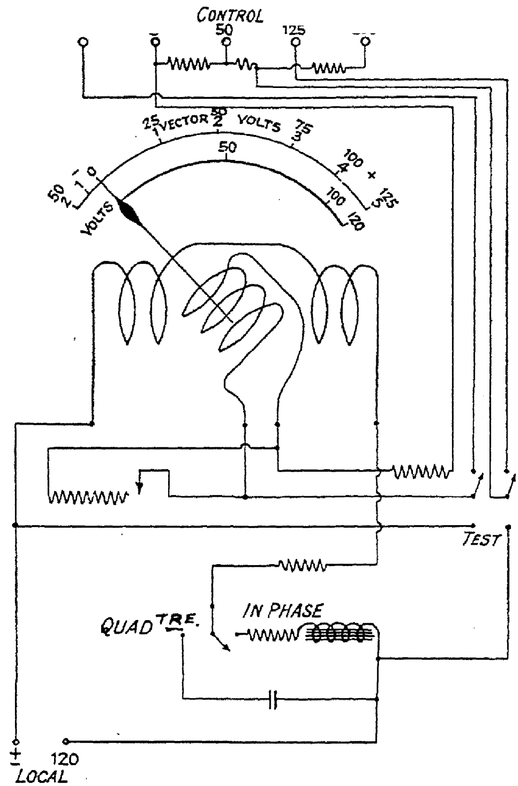


FIG. 55.—Circuit diagram of vector voltmeter.

circuit. With the second switch in the "In phase" position the component of voltage inphase with the local voltage will be indicated. With the switch in the quadrature position the component of voltage in quadrature with the local voltage will be indicated. The circuit is shown in Fig. 55.

This analysis of the two components is made by swinging

the phase of local current through 90° when the quadrature component is measured. Thus in its inphase position it measures like a wattmeter, and in the quadrature position it measures like a sine meter.

The instrument has found a variety of applications among which are the measurement of railway signalling circuits and the flexing of aeroplane propeller shafts.

Students' A.C. Potentiometer.—A simplified form of the co-ordinate a.c. potentiometer has been designed [6] for use upon the supply mains at one frequency. This instrument consists of two slide wires, one fed directly from a suitable secondary winding of a transformer and the other through a condenser from a tertiary winding. The volt drop upon the two wires is adjusted to be in quadrature by means of an inductance in the inphase slide-wire circuit. The instrument is standardized by means of a voltmeter across the primary of the transformer and a rheostat permits the adjustment of the voltage to a red mark upon the dial indicating the correct voltage. The absolute accuracy is about 2 per cent., but relative voltage measurements can be made to a higher precision than this.

A further development in student instruments is the Polar Co-ordinate Students' a.c. potentiometer. This instrument combines a phase shifter with the co-ordinate potentiometer so that the phase of both slide wires can be swung through any angle. Both slide wires are provided with current measuring instruments so that they can be adjusted to their correct value with ease. The phase-shifting transformer can be fed from a three-phase or two-phase source direct or from a phase-splitting circuit and a single-phase source. Fig. 56 (*a* and *b*) shows the potentiometer.

Detectors for use in A.C. Potentiometer Measurements.—In most cases the moving-coil type of vibration galvanometer is the most suitable form of detector since it has the advantage of good sensitivity combined with rapid damping. When the coil vibrates between its pole pieces an e.m.f. is generated in the coil which, by Lenz's electromagnetic law, tends to obstruct its own movement. This property of the moving-coil vibration galvanometer which causes it to come rapidly to rest is of great advantage in rapid measurements.

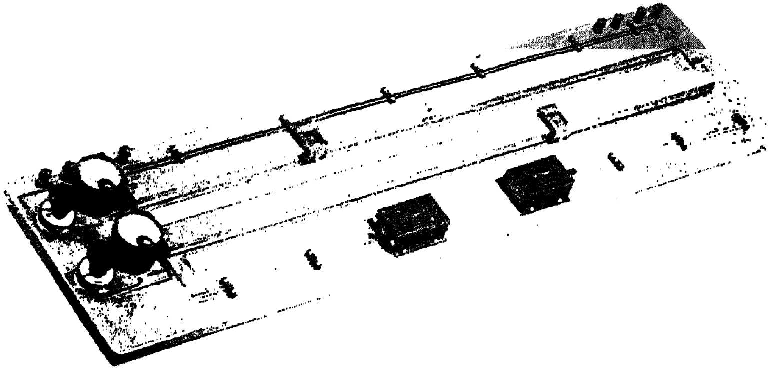


FIG. 56a.—Inphase and quadrature potentiometers—Students' a.c. potentiometer.

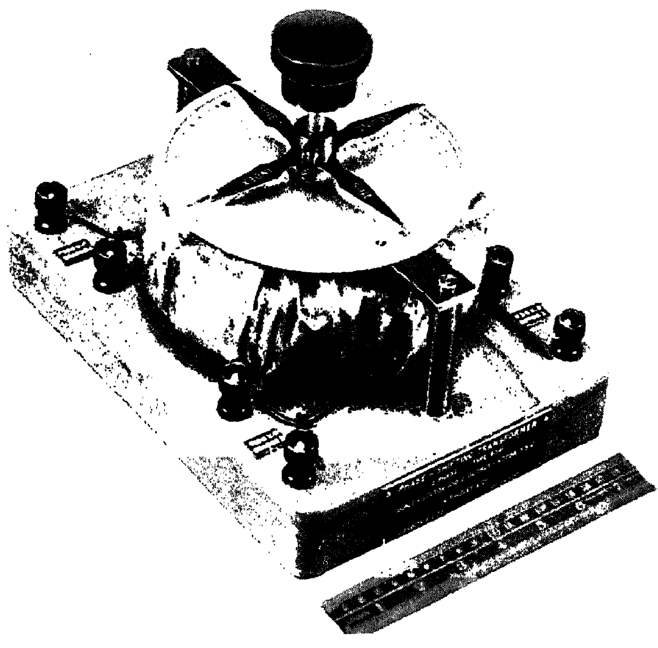
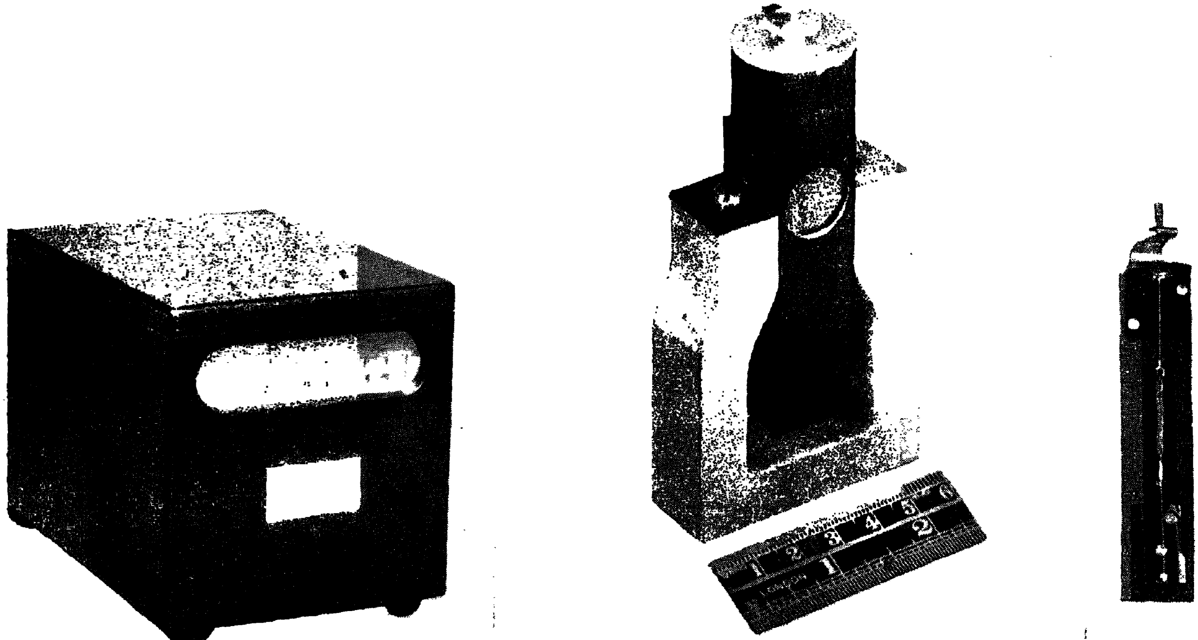


FIG. 56b.—Phase-shifting transformer—Students' a.c. potentiometer.



(a) (b) (c)

FIG. 57.—Vibration galvanometer.

(a) Complete unit. (b) The galvanometer. (c) The galvanometer movement.

[To face p. 95

Fig. 57 shows a self-contained vibration galvanometer suitable for use at one frequency.

It is a desirable feature that the galvanometer or any other detecting device used in making a balance, should take up its final position rather faster than the dials of the instrument can be operated. In the older moving iron types of vibration galvanometer this was quite impossible, as there is very little damping of the vibrating element, so that balancing was a tedious process and a pause had to be made after each adjustment to allow the vibration galvanometer to settle down to something near its final value in order to observe the effect of the adjustment made. With a suitable moving-coil vibration galvanometer this does not happen, and the galvanometer responds immediately to the adjustment of the circuit, so that it is possible to obtain a balance as quickly as the dials can be moved. In order to obtain this rapid response, it is necessary that the galvanometer should be acting into a circuit of suitable impedance. The galvanometer should generally be heavily shunted until the final accurate balance is made. Even when shunted with a resistance of a fraction of an ohm, the sensitivity of a good moving-coil vibration galvanometer is sufficient to allow a balance to a few millivolts to be made. The galvanometer can then be unshunted and the final balance made with the highest precision to which the circuit lends itself. A moving-coil vibration galvanometer working with a 1-metre scale distance can usually be obtained with sufficient sensitivity for two or three microvolts to be detected with ease. Where a higher degree of sensitivity is necessary, it may be advisable to make use of a valve amplifier in the galvanometer circuit (particularly if a very much higher degree of sensitivity is required and small fractions of a microvolt have to be measured). In choosing a valve amplifier for this purpose, it is essential that the amplifier should not pick up stray magnetic fields and so give rise to false indications. The design of suitable amplifiers requires considerable care; on account of their liability to be influenced by stray magnetic fields from transformers, resistance coupling, although not essential, is usually employed between stages, but as the galvanometer is of moderate impedance, it is almost essential to use a transformer in the output

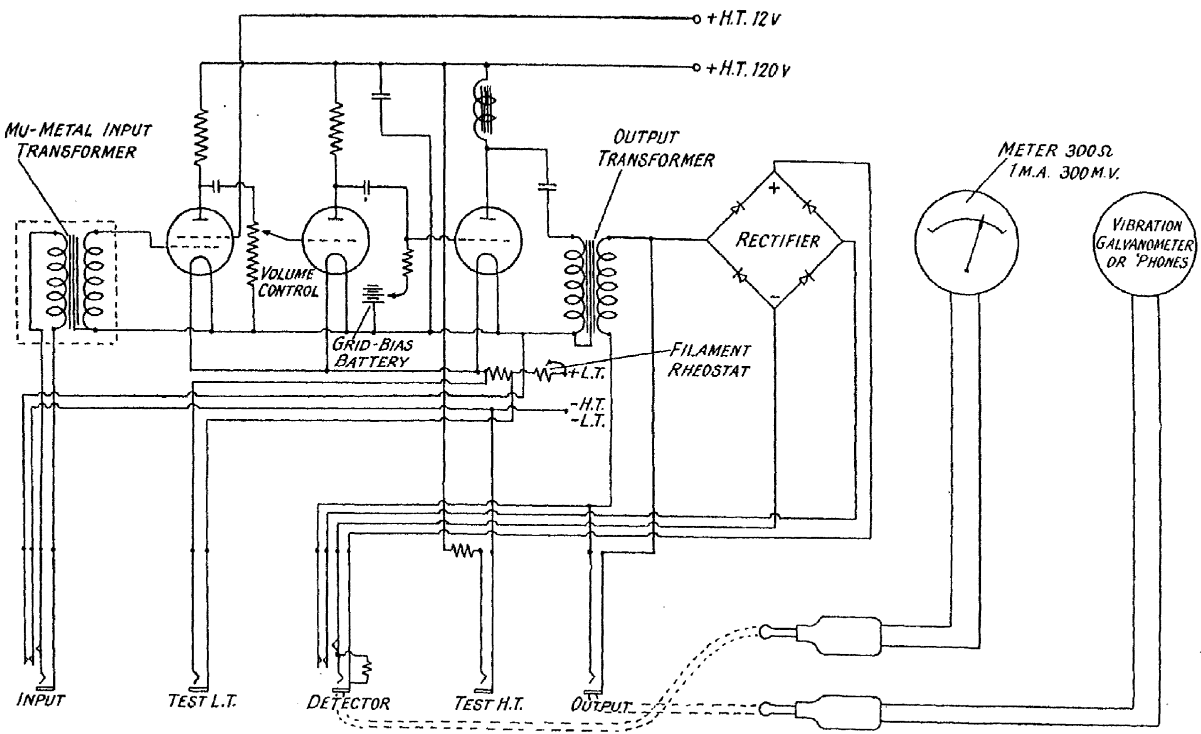


FIG. 58.—Out-of-balance amplifier for low-frequency supplies.

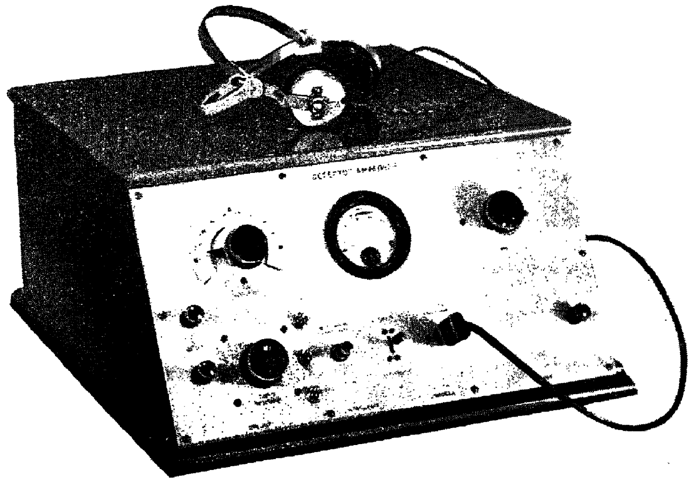


FIG. 59.—Valve amplifier for use in balance circuit of a.c. potentiometer.

[*To face p.* 97

stage, and often a transformer is necessary in the input stage in order to match the impedance of the amplifier with the potentiometer circuit to some degree of equality. Unless these circuits are reasonably matched, there will be a great loss of efficiency in the amplifier. In connexion with the input transformer, it is advisable to use a high permeability core (such as mumetal), because the voltages applied to the input side will be very small and the initial permeability of the iron in the input transformer should be as high as possible if these small currents are to be usefully transformed. A suitable circuit is shown in Fig. 58 and an illustration of the complete amplifier in Fig. 59. It is best for the whole unit to be self-contained with its own batteries as it is almost impossible to use a mains supply if the mains are at the same frequency as the measurement circuit for which the amplifier is being employed. It is quite possible to use a galvanometer with a rectifier after the last stage of the amplifier instead of the vibration galvanometer, but it should be remembered that in the a.c. potentiometer measurements, the presence of harmonics and any other stray frequencies will vitiate accuracy and, for this reason, for potentiometer work it is preferable to have a sharply tuned detector even at the output end of an amplifier. When work at higher frequencies is to be done, the tuning can be carried out in the amplifier circuit, in which case there is no objection to the use of a rectifier instrument providing it does read zero with zero input, but if there are any harmonics present a clean balance will be unobtainable. One of the chief difficulties in the use of an amplifier is the restriction that it imposes on earth capacitances and earth connexions in the galvanometer circuit. The whole circuit must be carefully considered from this point of view in order to see that the galvanometer circuit can be safely earthed on one side. If this cannot be done, then some form of isolating transformer, which allows and provides for sufficient earth screening, must be employed in the input stage of the amplifier. At frequencies above 200 cycles per second, a telephone is often the most convenient form of detector. The ear is able to distinguish the fundamental balance, should small harmonics be present. The sensitivity of the high-resistance telephone without amplification is usually such that about 100

microvolts can be readily detected. This depends, of course, upon the impedance of the telephones which should match the impedance of the circuit in which they act as nearly as possible. As low-resistance telephones cannot so readily be obtained as high-resistance ones, a matching transformer having a suitable ratio of transformation can be used. For balance work a high permeability core is required, but a very small core can be used since the energy to be transformed is minute. If, due to saturation of the transformer or the amplifier, the telephone does not indicate approximately the magnitude of the out-of-balance voltage, it is very difficult to find the balance condition, because the note amplitude does not decrease as balance is approached. When a high gain amplifier is used with the detector, it is essential to provide a means of reducing the sensitivity very much in the preliminary stages of obtaining a balance. This must be done at the input side by means of an attenuator or by cutting out the preliminary stages of the amplifier to make the response proportional to the out-of-balance voltage.

For high-frequency measurements a heterodyne detector amplifier is usually employed. This includes an oscillator, oscillating at such a frequency that a beat note is produced with the frequency at which the potentiometer is being worked. The resultant beat note is then amplified and taken to a pair of telephones. The circuit of such a detector amplifier is shown in Fig. 60; the actual instrument is similar in appearance to that illustrated in Fig. 59. It is important to keep the input impedance of the amplifier fairly low and to avoid stray induced potentials in this part of the circuit. This can be done by winding the "mixing" transformer in balanced toroidal form and using resistance coupling in the amplifier. (Coupling transformers are apt to pick up stray magnetic fields.) The purpose of the "mixing" transformer is to superimpose the beat frequency upon the measuring frequency before amplification. The input transformer must be screened and the amplifier separately screened and arranged so that the output circuit can be at "earth" potential. In cases where one side of the circuit cannot be earthed, a differential transformer can be employed in the detector and potential circuit, as in Fig. 70.

An A.C. Standard Cell.—This device is similar in prin-

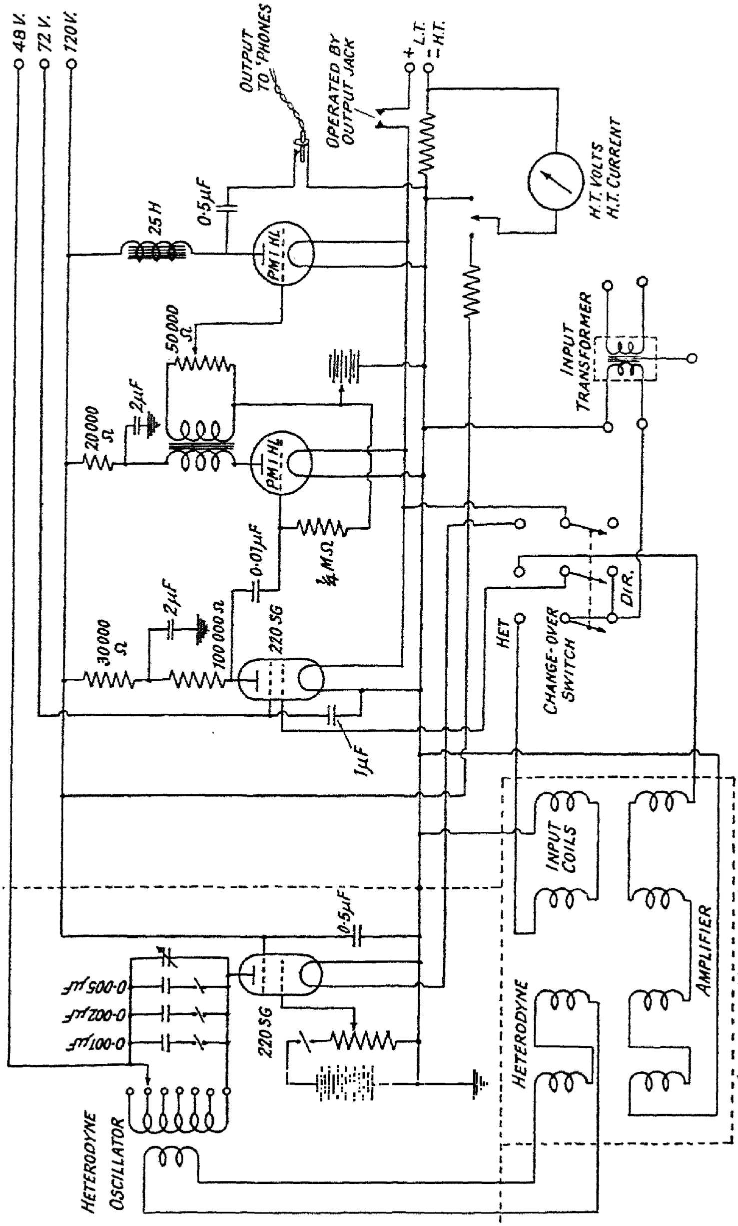


Fig. 60.—Out-of-balance amplifier for high-frequency supplies.

ciple to the differential thermal device adopted by Campbell to ensure the equality of the currents in the two parts of the Campbell-Larsen potentiometer, as explained on page 69.

In the a.c. standard cell two matched thermal converters, consisting of a number of Moll thermo-couples connected in series, with their active junctions arranged to be in thermal contact with an electrically insulated heating wire, are used to compare the heating effect of a direct and alternating current. The d.c. is passed through the heating wire, a standard resistance and a variable resistance. The current is adjusted so that the voltage drop across the standard resistance is equal to the e.m.f. of a standard cell. The a.c. is passed through a similar circuit. The circuit is shown in Fig. 61. When the effective or r.m.s. value of the a.c. in the second thermo-couple

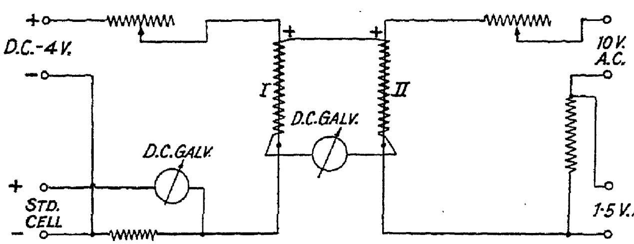


Fig. 61.—An a.c. standard cell.

is exactly equal to the d.c. in the first, the e.m.f.s induced in the thermo-couples are equal and in opposition and no current is passed through the galvanometer.

Under this condition the voltage drop across the standard resistance in the a.c. circuit must be some exact proportion of the standard cell voltage, and it is usually arranged to be 1·5 volts.

In use the a.c. standard cell would replace the dynamometer instrument, the potentiometer being standardized directly on a.c. instead of on d.c. as in the potentiometers described. At low supply frequencies this is not of great advantage, for the accuracy obtainable by the use of a dynamometer instrument is higher than that obtainable with a thermal converter and in addition the dynamometer serves the very useful purpose of showing immediately whether the potentiometer is off stan-

dardization. This latter point assumes considerable importance when measurements of high precision are being made. Thermal instruments are generally easily damaged by small overloads and are rather sluggish in action, so that their use is restricted to the higher frequencies where the dynamometer type becomes unsuitable owing to its high impedance and loss of accuracy due to eddy currents and self-capacitance in the windings. The dynamometer instrument is accurate up to 2,000 cycles per second.

NOTES AND BIBLIOGRAPHY.

[1] GALL, DOUGLAS C.: "A New A.C. Potentiometer," *Electrician,* 1923, xc, 360–1.

[2] GALL, DOUGLAS C.: "A Portable A.C. Potentiometer," *J. Sci. Instrum.,* 1925, iii, 202–4.

[3] GALL, DOUGLAS C.: "A Vector Voltmeter," *J. Sci. Instrum.,* 1933, x, 391–3.

[4] GALL, DOUGLAS C.: "An A.C. Potentiometer for Students," *J. Sci. Instrum.,* 1930, vii, 132–3.

[5] Patent Specification 334131.

[6] STEINMETZ, C. P.: "Theory and Calculation of Electric Circuits," pp. 280–1, 1917 (McGraw & Hill, New York).

[7] DRYSDALE, CHARLES V.: "The Use of a Phase-shifting Transformer for Wattmeter and Supply Testing," *Electrician,* 1908, lxii, pp. 341–3.

CHAPTER VIII.

USES OF THE A.C. POTENTIOMETER.

Calibration of Meters.—The calibration of an ammeter or a voltmeter is a simple example of the use of the a.c. potentiometer. For this purpose the process is almost identical with the d.c. calibration described in Chapter IV. It is essential that the instrument under test be supplied from the same source as the a.c. potentiometer, but a transformer, variable rheostat or choke may be interposed to control the current through the instruments.

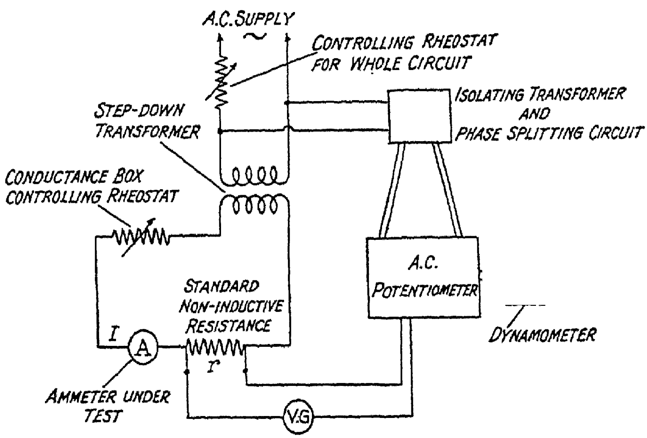


FIG. 62.—Circuit diagram for calibration of an ammeter.

Fig. 62 shows the schematic arrangement of an ammeter under test. Having set up the circuits and correctly adjusted the potentiometer, the current through the ammeter should be adjusted to some definite indication of the pointer. The volt drop upon the standard resistance should then be measured and the correct current passing through the ammeter will then be given by $I = e/r$ where r is the resistance value. It is usual to tabulate the results as shown in Table II in taking readings

TABLE II.

CALIBRATION OF WESTON SUBSTANDARD AMMETER, RANGE 0–5 AMPERES.

Ammeter Reading.	Potentiometer Readings.		Magnitudes.		Current (Amperes) Average.	Calibration Figure Adopted (Amperes).
	Forward.	Reverse.	Forward.	Reverse.		
1·00	0·1961 + j0·0316	− 0·1969 − j0·0318	0·19863	0·19945	0·9952	
	0·1962 + j0·0316	− 0·1968 − j0·0317	0·19872	0·19933	0·9951	0·995
1·50	0·2967 + j0·0458	− 0·2964 − j0·0457	0·30021	0·29990	1·5003	
	0·2967 + j0·0458	− 0·2960 − j0·0454	0·30021	0·29950	1·4993	1·500
2·00	0·3960 + j0·0558	− 0·3960 − j0·0569	0·39996	0·40008	2·0002	
	0·3961 + j0·0554	− 0·3958 − j0·8567	0·39996	0·39984	1·9995	2·000
2·50	0·4661 + j0·1798	− 0·4660 − j0·1801	0·49957	0·49956	2·4978	
	0·4660 + j0·1795	− 0·4664 − j0·1801	0·49934	0·49997	2·4983	2·498
3·00	0·5698 + j0·1864	− 0·5701 − j0·1868	0·59868	0·59929	2·9974	
	0·5694 + j0·1862	− 0·5693 − j0·1868	0·59908	0·59847	2·9938	2·995
3·50	0·6732 + j0·1904	− 0·6750 − j0·1834	0·69961	0·69948	3·4977	
	0·6733 + j0·1903	− 0·6747 − j0·1837	0·69967	0·69926	3·4972	3·497
4·00	0·7820 + j0·1660	− 0·7828 − j0·1634	0·79942	0·79950	3·9973	
	0·7820 + j0·1660	− 0·7824 − j0·1629	0·79942	0·79918	3·9965	3·997
4·50	0·8862 + j0·1554	− 0·8834 − j0·1699	0·89972	0·89960	4·4982	
	0·8862 + j0·1559	− 0·8818 − j0·1805	0·89980	0·90005	4·4998	4·499
5·00	0·9877 + j0·1539	− 0·9881 − j0·1527	0·99962	0·99984	4·9986	
	0·9883 + j0·1529	− 0·9880 − j0·1524	1·00008	0·99968	4·9994	4·999

The variation of position of the current vectors, with respect to the potentiometer, between 2·0 and 2·5 amperes readings, is due to the use of a different transformer in the ammeter supply circuit, but it does not affect the magnitude.

For the purpose of comparison, a d.c. potentiometer calibration of the ammeter is given below together with the a.c. calibration.

TABLE III.

Ammeter Reading.	D.C. Calibration.	A.C. Calibration. (From Table II.)
1	0·997	0·995
2	1·997	2·000
3	2·997	2·995
4	4·000	3·997
5	4·999	4·999

with current supply to the circuit under test reversed in order to eliminate induced e.m.f.s between the current and the potentiometer circuits.

The above results were measured upon the co-ordinate type of potentiometer, but for this type of calibration the polar or Drysdale type of a.c. potentiometer is preferable, because the phase of the potentiometer can be orientated to agree with the volt drop upon the standard resistance. Thus no vector calculation will be required, beyond dividing the measured value of e by the value of r, but the dynamometer must be sufficiently sensitive to give the desired accuracy. This necessitates a reflecting type instead of a pointer type of dynamometer.

Where the co-ordinate type of potentiometer is used the volt drop on the non-inductive resistance by which the current is determined will be measured in two components in the form $e = a + jb$. The resultant value will be $\sqrt{a^2 + b^2}$ so that this calculation is necessary in order to determine the current. The actual value of this current will be $I = 1/r\sqrt{a^2 + b^2}$ when r is the value of the resistance and a and b the inphase and quadrature components respectively.

In the case of calibrating a voltmeter the circuit would be arranged as in Fig. 63. It is sometimes convenient to use a potential divider to supply the voltmeter in addition to the controlling rheostat.

The voltage across the voltmeter terminals would be adjusted by any convenient means and the volt drop upon the volt ratio box measured by means of the a.c. potentiometer.

In the case of the polar type of potentiometer the voltage will be read off directly when balanced because the phase of the potentiometer will agree with that of the volt drop. The actual voltage across the terminals of the voltmeter will be eN where N is the ratio of the volt ratio box and e the measured volt drop.

In the case of the co-ordinate type of a.c. potentiometer the volt drop will be measured in two components $e = a + jb$. The actual magnitude of the volt drop will be $\sqrt{a^2 + b^2}$ so that the terminal voltage will be $N\sqrt{a^2 + b^2}$.

It is important that the volt ratio box be connected exactly

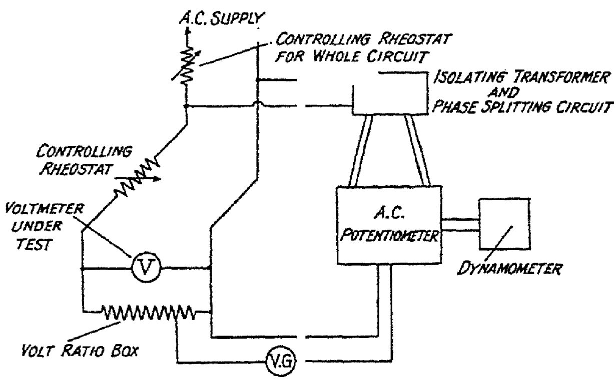


FIG. 63.—Circuit diagram for calibration of a voltmeter.

across the actual terminal points between which the calibration is required. The resistance of the volt ratio box should not be too high or the sensitivity of the galvanometer will be much reduced. About 50 to 100 ohms per volt is a suitable figure for this type of measurement.

It is usual to prepare a calibration table for the voltmeter calibration as shown in Table IV.

Mechanical hysteresis in the instrument under test can be determined by taking measurements with the current increasing and then with the current decreasing, without allowing the pointer to overswing at each calibration point. When the rising and falling calibration figures are plotted against scale reading, a loop will be formed giving the hysteresis of the

TABLE IV.

CALIBRATION OF SUBSTANDARD WESTON VOLTMETER, RANGE 0–150 VOLTS.

Voltmeter Reading.	Potentiometer Readings.		Magnitudes.		Voltage.	Calibration Adopted (Volts).
	Forward.	Reverse.	Forward.	Reverse.	Average.	
60	0·5926 + j0·1064	− 0·5922 − j0·1057	0·60208	0·60156	60·182	
	0·5923 + j0·1066	− 0·5922 − j0·1054	0·60232	0·60151	60·192	60·19
80	0·7887 + j0·1399	− 0·7884 − j0·1390	0·80101	0·80056	80·078	
	0·7892 + j0·1398	− 0·7886 − j0·1392	0·80150	0·80079	80·114	80·10
100	0·9869 + j0·1712	− 0·9856 − j0·1710	1·00164	1·00032	100·098	
	0·9868 + j0·1710	− 0·9854 − j0·1704	1·00151	1·00003	100·077	100·09
120	1·1830 + j0·2084	− 1·1825 − j0·2088	1·2012	1·2008	120·10	
	1·1834 + j0·2083	− 1·1828 − j0·2090	1·2016	1·2011	120·135	120·12
150	1·4766 + j0·2697	− 1·4770 − j0·2653	1·5010	1·5006	150·08	
	1·4762 + j0·2696	− 1·4768 − j0·2667	1·5006	1·5007	150·065	150·07

instrument. The pointer will take up a mean position in the loop when the instrument is gently tapped. This is usually taken as the calibration value.

Calculation of Resultant Values.—It will be evident that for the simple calibration of voltmeters and ammeters the co-ordinate measurement necessitates a calculation for the determination of the measured quantity. When measurements are required of the highest order of accuracy of which the potentiometer is capable, it is necessary to use seven-figure logarithms or for preference a calculating machine to find the square root of the sum of the squares of the component parts of the measured voltage. When a lesser accuracy is required this calculation can be performed easily by means of the standard pattern slide rule. The procedure is as follows:

If the measured voltage in the co-ordinate form $e = a + jb$ is $e = 1{\cdot}253 + j0{\cdot}427$ the magnitude of e will be

$$\sqrt{1{\cdot}253^2 + 0{\cdot}427^2}.$$

Set 427 on the C scale to 1,253 on the D scale. Over the right index of the slider or B scale read upon A scale the value 8·62. To this add 1 by sliding the index to 9·62 on A scale. Under 427 on C scale read 1·325 on D. This is the resultant value of the root of the sum of the two squares.

This process consists of the following steps:

Vector value $e = a + jb$

Magnitude $e = \sqrt{a^2 + b^2}$

First setting gives a/b on scales C and D

Second " " a^2/b^2 on scale A

Third " " $a^2/b^2 + 1$ on scale A

Fourth " " $b\sqrt{a^2/b^2 + 1}$ on Scale D.

Care must be taken in adding 1 in the third step. The correct position of the decimal point must be kept in mind. This is facilitated by dividing always by the smaller of a or b. There is a special type of slide rule known as the Davis-Grinstead rule for making these and similar calculations with great facility.

Sources of Error.—When measurements of the magnitude of a voltage are made on the co-ordinate type of potentiometer

two possible sources of error are introduced. The first is that the inphase and quadrature voltages of the potentiometer itself may not be in true quadrature due to impurities or induced e.m.f.s in the mutual inductance. The second is that the frequency of the supply voltage to both potentiometer and circuit under test may vary slightly from the value used when standardizing the potentiometer.

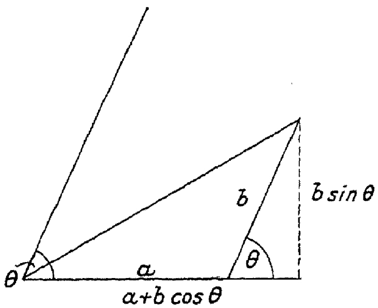


FIG. 64.—Departure of axes from true quadrature.

If the angle of phase difference θ between the inphase and quadrature voltages of the potentiometer differs slightly from 90°, then the true value of any voltage $e = a + jb$ will be $e = a + b\cos\theta + jb\sin\theta$, as can be seen from Fig. 64. The measured magnitude of the voltage V_m will be $\sqrt{a^2 + b^2}$ whilst the true value V_T will be $\sqrt{a^2 + b^2 + 2ab\cos\theta}$.

Then $$V_T{}^2 - V_m{}^2 = 2ab\cos\theta$$
$$\therefore\ (V_T - V_m)(V_T + V_m) = 2ab\cos\theta$$

and since V_T and V_m are nearly equal the percentage error

$$100\,(V_T - V_m)/V_T \doteqdot 100\,ab\cos\theta/V_m{}^2$$
$$\doteqdot 100\,b/a\cos\theta/(1 + b^2/a^2).$$

The error will have a maximum value when $b/a = 1$ or the vector voltage measured is at 45° to the co-ordinate system of the potentiometer. If the required order of accuracy of measurement is 1 part in 2,000

$$100\,b/a\cos\theta/1 + b^2/a^2 = 0{\cdot}05$$

and when $b/a = 1$ $\qquad \cos\theta = 0{\cdot}001$

and $\qquad \theta = 89°\ 57'.$

That is, to measure a voltage to the highest order of accuracy of which the potentiometer is capable, the error in quadrature of the potentiometer itself must be less than 3 minutes of a degree under, the most unsuitable condition of measurement, that is with the voltage to be measured at 45° to the inphase

potentiometer voltage. This error will obviously be zero when either a or b is zero. If the error of quadrature is 1° (a large error) and b is 10 per cent. of a, the error will be only 0·01 per cent.

If the frequency of the supply differs from the value at which the potentiometer is standardized, a proportionate correction is necessary to both components of the measured voltage in the Larsen type of potentiometer, but to the quadrature component only in the Gall type.

In practice when using the frequency of the grid-controlled mains for the supply (see Chapter XII) a variation of frequency of 0·2 per cent. may occur. Assume that this is higher than the specified value and that the magnitude of the voltage is required to 1 part in 2,000 as before. The correction factor to be applied to the quadrature component b of the measured voltage $V_m = a + jb$ is then 1·002. The corrected value of the measured voltage is therefore $V_T = a + j1{\cdot}002b$.

Then $$V_T/V_m = \sqrt{\frac{a^2 + 1{\cdot}002b^2}{a^2 + b^2}} = 1{\cdot}0005$$

from which $b/a = 0{\cdot}57$. Hence if the quadrature component of the measured voltage is less than 57 per cent. of the inphase component, the error due to a frequency variation of 0·2 per cent. is less than 1 part in 2,000 in the calculation of the magnitude.

Of the two sources of error discussed it will be seen that the former is likely to lead to the larger errors, for it is usually a simple matter to arrange that the quadrature component shall be less than 57 per cent. of the inphase component of the measured voltage by ensuring that the test circuit is not highly reactive. It will be observed that this condition was fulfilled in the experimental results tabulated on pages 103 and 106.

It will be evident that both of these errors are avoided theoretically by the use of the polar type of a.c. potentiometer. A phase-shifting transformer is often used to supply the co-ordinate type of potentiometer so that the inphase potentiometer can be turned into any desired phase relationship with respect to the supply, in order to eliminate the quadrature component from the measurement when this is desirable, as in the cases

just considered, but the phase-shifter is also affected by fluctuation in precisely the same way, when supplied from a single-phase source.

Fig. 65 shows the arrangement for supplying the co-ordinate a.c. potentiometer from a three-phase source through a phase-shifting transformer. This gives the advantage of both types of potentiometer, and is often the means of eliminating some tedious calculations when current voltage or flux only are being measured. Calculation is unavoidable when power is measured.

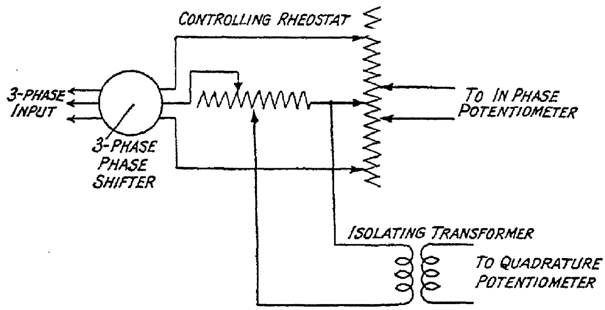


FIG. 65.—Supply to co-ordinate a.c. potentiometer from three-phase source.

Practical Details of Current Measurements.—The calibration of ammeters in the way outlined above is usually a simple matter. The chief practical difficulties are encountered in obtaining the desired currents under sufficient control. To obtain high accuracy in the current measurement the standard non-inductive resistance inserted in series with the ammeter should be of such a value that the volt drop is about 1 volt for full-load current, that is, for calibrating a 5-ampere meter a resistance of 0·2 ohm would be of a convenient value. This would give a volt drop of 0·1 volt at one-tenth of full load which is easily determinable to a precision of 1 millivolt or to an accuracy of 1 per cent. The precision upon this basis would be 0·1 per cent. at full load.

For current measurements at low frequencies the non-inductive requirements of the resistance standards are not very critical as long as the magnitude of current and not the phase is required. The magnitude error in the volt drop will be the

difference between the impedance and the resistance. This will be given by:

$$\Delta Z = Z - r$$

where $$Z = \sqrt{r^2 + \omega^2 L^2}$$

where r is the resistance, L the inductance and $\omega = 2\pi$ times the frequency. The value $\frac{L}{r}$ is the time constant T

so that $$\Delta Z = r(\sqrt{1 + \omega^2 T^2} - 1)$$

if $\omega^2 T^2$ is small $$\Delta Z = \tfrac{1}{2} r \omega^2 T^2.$$

A time-constant of 10^{-5} is very large for a non-inductive resistance of good manufacture so that at 50 cycles the impedance error of such a resistance would only be $\Delta Z = 5 \times 10^{-6} r$ or an error of 5 parts in a million of its resistance value.

The phase error will be given by

$$\varphi = \tan^{-1}\frac{\omega L}{r}$$

$$= \tan^{-1}\omega T$$

If $\frac{\omega L}{r}$ is small $\tan\ \omega T = \omega T = \varphi$ radians

$$\therefore\ \varphi = 57{\cdot}3\omega T \text{ degrees}$$

That is $\varphi = 0{\cdot}18^\circ$ in previous example.

This would be a serious phase error in some kinds of measurements such as in calibrating a current transformer, and for such purposes the time-constant of the resistances should be of the order of 10^{-7}.

Non-inductive shunts are made having large current-carrying values. Fig. 66 shows one for 1,000 amperes. This is oil- and water-cooled. The N.P.L. type of non-inductive shunt [1] shown in Fig. 67 is air-cooled, which makes it much more bulky for the same rating, but avoids the disadvantage of oil- and water-cooling. Further, the time-constant of the N.P.L. type can be adjusted to either side of zero, so that the non-inductive perfection is restricted only by the precision with which the phase angle can be measured. This type is not made above 500 amperes.

For very large values of alternating current a special type

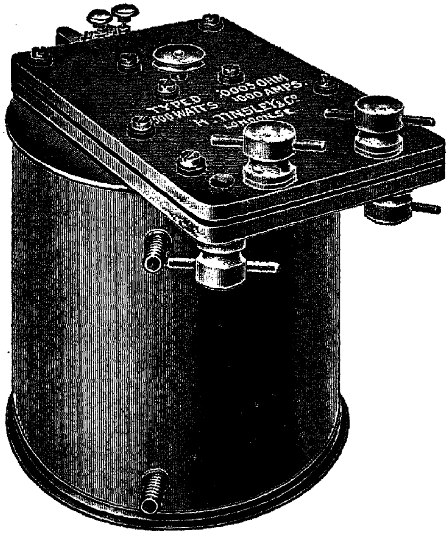


FIG. 66.—Standard resistance for 1,000 amperes—oil-cooled. (Drysdale-Tinsley.)

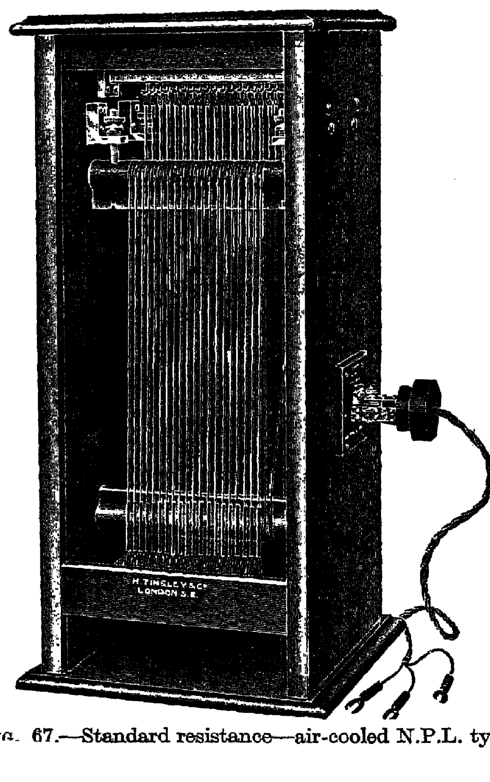


FIG. 67.—Standard resistance—air-cooled N.P.L. type.

of transformer shunt can be used. This consists of a current transformer with a non-inductive shunt in the secondary circuit. The volt drop is measured upon the shunt exactly as in the case in which the whole current passes through the resistance. This type of shunt is more economical for ratings exceeding a few thousand amperes, and has the advantage of an isolated testing circuit, but it cannot be standardized upon d.c.

In controlling the current through the circuit under test, the apparatus chosen depends upon the magnitude of the current. A well-made step-down transformer with a few large current-carrying secondary turns is convenient, as it can effectively isolate the secondary circuit from capacitive or leakage effects associated with the higher voltage of the supply. The actual measuring circuit can be almost at earth potential.

A conductance box in the current circuit is a much better controlling device than a resistance box. In a conductance box all the coils are in parallel so that the larger the current the more resistance coils there are to carry it. The same applies to the switches of a conductance box, which being in parallel have a much less variable contact effect in the circuit.

Conductance boxes can be obtained for controlling circuits from a few milliamperes up to several hundred amperes. For example, for use upon a 2-volt supply a conductance box of six dials of the values,

10×50	mhos.
10×10	,,
10×1	,,
$10 \times 0{\cdot}1$	,,
$10 \times 0{\cdot}01$	,,
$10 \times 0{\cdot}001$	,,

would control currents from 2 milliamperes up to 1,000 amperes in 2-milliampere steps. The 10×50 mhos dial would most probably be a plug dial. The type of switch used for the other dials is shown in Fig. 68.

To obtain a measurement of high precision it is necessary to leave both the test circuit and the potentiometer on circuit until a steady state has been reached. If this is not done, not

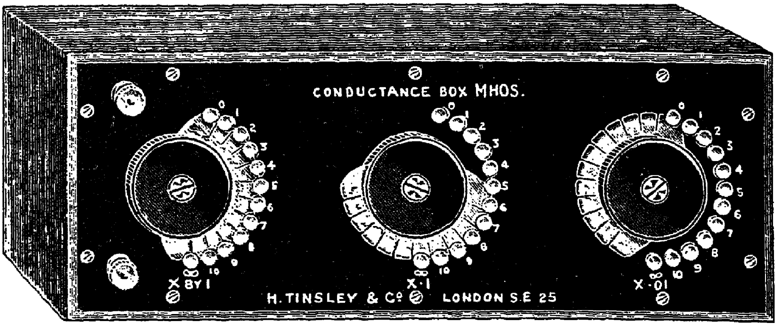


FIG. 68.—Conductance box.

only will the standardization of the potentiometer not be maintained but the current required to produce a certain indication on the instrument under test may vary, by an amount which can be measured on the potentiometer.

Voltage Measurements.—Low voltages to the value of 1·8 in magnitude can be measured directly on the potentiometer. Above this value a volt ratio box is used, exactly as in the case of the d.c. potentiometer. It is important that the potentiometer be connected to the "earth" side of the circuit, the volt ratio box being connected to the voltage under test in such a way that this can be done. The resistance of most volt ratio boxes is subdivided so that the tapping for the low voltage is at one end of the resistance. This end must be made the "earth" side. In some circumstances, this cannot be done, as, for example, when both sides of the high-voltage circuit are at a high potential above earth. A centre tapped volt ratio box may then be used. In some cases it is necessary to adjust the position of the potentiometer tapping to the appropriate position in the volt ratio box to maintain this tapping sensibly at earth potential. This is best done by the use of separate resistors.

When the potentiometer is connected to a high-voltage circuit, precautions must be taken to protect the user. This is usually done by carefully earthing the screen of the instrument and keeping it as far as possible from the high-voltage supply. If the circuits are properly arranged, there will be no high voltages on any of the terminals. If there are, there may

be serious danger of damaging the instrument when switching over, due to arcing across the selector switch, although this breaks between each position. Apart from these dangers, there will be capacitance and leakage currents in the potential circuits if the potential of the circuit under test differs largely from that of the potentiometer.

These currents will flow in the same direction along both potential leads to the potentiometer. Since the detector will be in one of the leads, this will detect their presence and the condition of balance will be a false one. This should be clear from Fig. 69.

The potentiometer should be balancing the difference of potential between the point A and B due to the current flowing round the circuit, but since the points A and B are at a high potential above earth, capacitive currents will flow into the potentiometer and to earth through its capacitance and insulation. These currents will be practically the same in both potential leads because of the relatively small difference of potential between A and B. The volt drop in the potential leads will be proportional to the impedance, and since the detector is normally in one lead only, there will be a volt drop on this which will be indicated by the detector. It may be possible to balance this volt drop by some adjustment of the potentiometer, but this will give an entirely false condition of balance, as it will include the volt drop on the detector, as well as the volt drop it is desired to measure between A and B. This condition of the circuit frequently leads to conditions which cannot be balanced at all when the volt drop due to the capacitive current is larger than the potentiometer range.

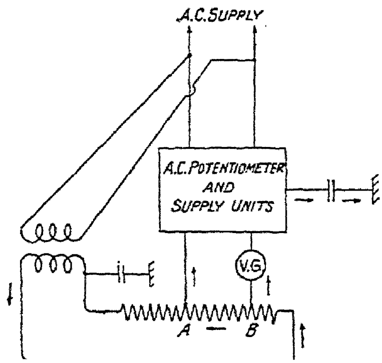


FIG. 69.—Capacitance and leakage currents in potential leads.

The detector, usually a vibration galvanometer, is generally

provided with a shunt to control the deflexion in the preliminary stages of balancing, and it is sometimes noticed that the balance setting of the potentiometer varies according to the position of the shunt. This is due to the change in the volt drop with the change in the resistance. The shunt may also introduce stray potentials if it is inductive and a stray magnetic field is present.

Whenever the former effect is present, the potentiometer readings are useless and the circuit must be modified to cure it. To test for its presence, disconnect alternately one side only of the supply to the potentiometer circuit and common the potential leads at either A or B. If the detector shows any deflexion then it must be due to capacitive or leakage currents of this nature.

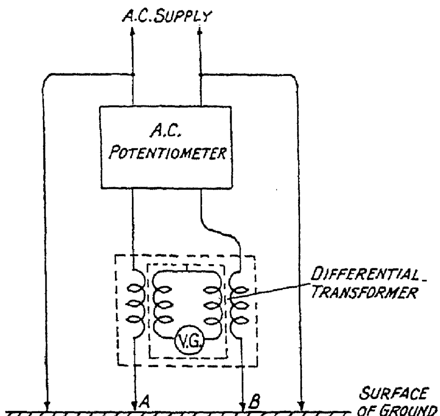


FIG. 70.—Use of differential transformer in potential leads.

The best remedy is to arrange one of the points A or B to be at the same potential as the potentiometer.

A method which is very successfully employed by the author in connexion with geophysical measurements, where large potential differences are unavoidable, is to use a differential transformer in the potential leads, as shown in Fig. 70.

The capacitive currents in the two potential leads balance each other since they are in the same direction, but the circulating current due to the volt drop between A and B is in opposite directions through the windings and so affects the detector in the balanced secondary circuit. The windings are screened and carefully balanced for impedance.

For high-voltage measurements, capacitance potential dividers may be used instead of resistance potential dividers. It is easier to construct a high-voltage condenser than a high-

voltage resistance potential divider, because of the capacitive effects in the resistance. These capacitive effects may completely alter the potential distribution from what would be expected from the resistance ratio. To overcome this, potential dividers are sometimes constructed in a number of relatively small screened units in series.[2-3] The screen of each unit is connected to a second potential divider and raised to the appropriate potential, so that it is at the mean potential of the unit it contains. In this way, the capacitive currents are limited and an even potential gradient along the divider is maintained.

This elaborate construction is expensive and the capacitance divider, consisting of a high-voltage small-capacitance condenser in series with a large-capacitance low-voltage condenser on the "earth" side, is simpler. The potential leads are taken to the large condenser and should be screened to prevent electrostatic effects. The capacitance of these leads will be added to that of the condenser to which they are connected, and this may require consideration in the true ratio of the potential division between the high-voltage and low-voltage condenser. Furthermore, the insulation must be of a very high order and the losses small, or these will seriously affect the ratio.

Potential transformers can be used as volt ratio boxes where it becomes necessary to isolate the apparatus from the supply. The normal 110-volt secondary winding can be connected to a volt ratio box of this range and the whole combination used in the same way as a simple volt ratio box. The ratio and phase-angle errors of the transformer itself can be corrected in the volt ratio box by modifying the ratio and by the introduction of reactance of the right value.

NOTES AND BIBLIOGRAPHY.

1 Spilsbury, R. S. J., and Arnold, A. H. M.: "Some Accessory Apparatus for Precise Measurements of Alternating Current," *J. Instn. Elect. Engrs.*, 1930, lxviii, 889–97.

2 Orlich, E., and Shultze, H.: "A Voltage Divider for H.T. Measurements," *Archive für Electrotechnik*, V. i, p. 1, 1912.

3 Davis, R.: "The Design and Construction of a Shielded Resistor for High Voltages," *J. Instn. Elect. Engrs.*, 1931, lxix, No. 416.

CHAPTER IX.

USES OF THE A.C. POTENTIOMETER (*continued*).

The Measurement of Power and Force.—The energy flowing in a circuit can be determined from the product of the current and the voltage. If these two quantities are in phase with each other, their product will give the power, but if the current and voltage are in quadrature with each other, the product will be the reactive volt-amperes. Both power and reactive volt-amperes are double frequency quantities of the dimensions of energy per second. The power derived in this way is the rate at which energy is being transformed into some other form, such as heat or mechanical work, and the reactive volt-amperes is the rate at which energy is stored in the field of the circuit in which the current and voltage exist. This property of the circuit of storing energy usually plays an important part in the behaviour of the apparatus with which it is associated. The mechanical forces appearing in electromagnetic or electrostatic apparatus are directly related to the change in the storage of energy in their circuits,[1] so that measurements of this form of energy or reactive power are as important as the measurements of the power which is being dissipated or transformed into some other form. The energy which is stored in the electric field is available to do work if that field is changed, so that the measurement of the energy stored in a magnetic field before and after some event at once gives the amount of work done by this change. The energy in any circuit will be given by integrating the instantaneous product of the current and volts, with respect to time, *i.e.*,

$$\text{the energy } w = \int eidt.$$

The value of the voltage e in a circuit of inductance L will be

$$e = \mathrm{L}\frac{di}{dt}$$

It follows, therefore, that the energy in the circuit will be

$$w = \int Li\,di$$

$$\text{or } w = \frac{i^2 L}{2} \text{ joules.}$$

This is the well-known relationship for the energy stored in an inductance. If i is the effective value of an a.c. the energy will be pulsating between zero and a maximum value of i^2L at double frequency since i^2 is double the frequency of i. The mean stored energy will, therefore, be $\frac{i^2L}{2}$ joules.

The effective value of the reactive voltage in quadrature with, and due to the current, will be $e = 2\pi fLi$ so that it follows that the mean stored energy will be

$$\frac{ei}{4\pi f} \text{ joules}$$

where e and i are the voltage and current in quadrature with each other. That is ei is the reactive volt-amperes of the circuit which can at once be converted into joules.

In general, the work done in any electromagnetic device is given by the change in the reactive volt-amperes such that the force in grammes

$$F = \frac{10^7}{4\pi fg} \cdot \frac{d(ei)}{dl}$$

where $\frac{d(ei)}{dl}$ is the rate of change of reactive volt-amperes with respect to position l, g is the acceleration due to gravity equal to 981 cm. per second^2 and f the frequency.

By measuring the change in the reactive volt-amperes with change in position, the mechanical forces can be derived from the electrical measurements.

If the current and voltage are measured at the terminals of an a.c. electromagnet at various positions of the armature, the reactive volt-amperes will vary according to the armature position. The force exerted upon the armature will depend upon the rate of change of the reactive volt-amperes with position. It is most convenient in actual measurement to keep either the voltage or the current at a constant value, unless the

actual working condition of the electromagnet involves changes in both of these, due, for example, to line resistance or other impedances. If the current is $I = a_1 + jb_1$ and the voltage is $E = a_2 + jb_2$, measured at the magnet winding, the reactive volt-amperes will be $P_j = a_1b_2 - a_2b_1$. This is dealt with more fully in Chapter XIII. This is a simple numerical value for any position of the magnet armature. If P_j is plotted in a curve against the measured position of the armature the pull of the magnet in grammes at any position will be given by the slope of the curve, multiplied by $\frac{10^7}{4\pi fg}$.

Reduced to pounds the force is

$$F = \frac{1{\cdot}79}{f} \cdot \frac{dP_j}{dl} \text{ pounds.}$$

The force is pulsating at double frequency between zero and twice the mean value given by the above formulæ.

The same method of determining mechanical forces from purely electrical measurements can be applied to many electromagnetic devices such as pointer instruments or relays. The armature of the pointer must be held mechanically during the electrical measurement. In the case of the electromagnet, this can be done by fibre separators of various thicknesses placed between the pole pieces and the armature.

The Measurement of Power.—To measure the power in a circuit by means of the a.c. potentiometer, the current and voltage must be measured in the way described in Chapter VIII. If these two quantities are found to be $I = a_1 + jb_1$ and $E = a_2 + jb_2$ where a and b are the inphase and quadrature components respectively, the power will be given by $P = a_1a_2 + b_1b_2$ watts. (See Chapter XIII.)

That is to say, by the sum of the products of the two components, which are in phase with each other. The current a_1 is in phase with the voltage a_2 and the product is power. Similarly, the only voltage in phase with b_1 is b_2 and the product is more power.

The voltage b_2 is in quadrature with the current a_1 so that the reactive volt-amperes represented by this product will be a_1b_2. In addition there will be the reactive volt-amperes due

to a_2b_1, but the sign of this product will be reversed so that the total reactive volt-amperes will be

$$P_j = a_1b_2 - a_2b_1.$$

These products cannot be derived from the algebraic multiplication of the two single-frequency quantities a_1 and jb_1 and a_2 and jb_2, because the product of two sine waves is a double frequency sine wave so that the operator j representing 90° in the single frequency represents 180° in the double frequency. It must be remembered that the vector notation is a purely symbolic method of representing phase relationships in single-frequency quantities.

Example of a Power Measurement.—As an example of a power measurement, the calibration of a wattmeter can be

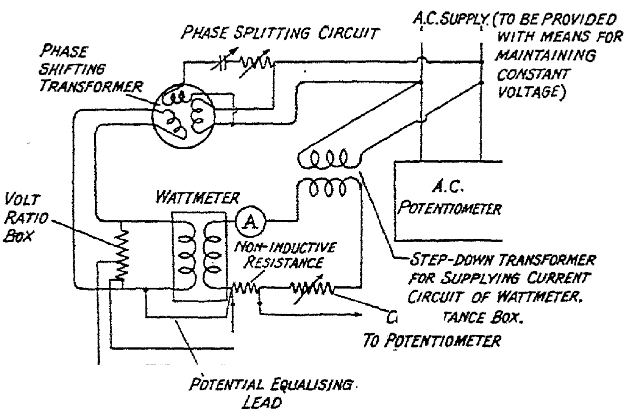


FIG. 71.—Circuit diagram for calibration of a wattmeter.

considered.[2] When it is desired to calibrate a wattmeter on various power factors, some means of producing these must be available. The most convenient arrangement is to have a separate supply for the current and voltage circuits with a phase-shifter supplying the latter. Fig. 71 shows the scheme.

The current circuit can be of very low voltage and the watts dissipated in the load will then be quite small. The current is measured by the volt drop in the standard resistance in the

current circuit and the voltage by the volt drop in the volt ratio box in the voltage circuit supplied by the phase-shifting transformer. An ammeter in the current circuit is convenient for providing a quick indication of approximate values and to prevent accidental overloading.

Unity power factor can be found approximately by turning the phase-shifter until no deflexion is indicated upon the wattmeter and then turning the phase-shifter through 90°. Other power factors can be estimated in the same way. Having adjusted the load current to a suitable value, such as $\frac{1}{10}$ of the full load, and turned the phase-shifter to unity power factor, the current and voltage are measured and recorded as shown in Table V.

The phase-shifter should then be turned to the zero power factor position and the wattmeter tested under this condition. This is usually a very severe test, and one for which the co-ordinate type of potentiometer is particularly suitable. In the polar type where the power must be derived from the product of the current and volts and the cosine of the angle between them, it will be obvious that, as the angle increases, the slightest error in the knowledge of the angle will result in a large error in the calculated power. Unfortunately, it is difficult to measure the angle very precisely with the polar type of potentiometer and an error of 0·2° near zero power factor will represent the difference between 3·5 per cent. of full power and zero. It is because the co-ordinate potentiometer does not require to measure the angle for any calculation that it is superior to the polar type in power measurements, although less convenient (but not less accurate) in current and voltage measurements. It is, of course, essential that the two potentiometers are accurately at right-angles, but the means of standardization against the mutual inductance facilitates a very accurate setting for this condition. The effect of an inclined co-ordinate system can be investigated as follows:

If the b axis departs from true quadrature with respect to the a axis by a small angle such that the angle between the axes is θ degrees, the true value of any vector will be $(a + b \cos \theta) + jb \sin \theta$ instead of the apparent value $a + jb$ (see Fig. 64). Thus in the calculation of power represented

TABLE V.

CALIBRATION OF A WATTMETER.

Manufacturers . . Weston Electrical Instrument Corporation.
Type . . . . Dynamometer.
. . 0·5 amp. 150 volts (*i.e.*, 750 watts).
Scale . . . Uniform—diagonally divided to 1 watt.

Instrument Reading. (1)	Potentiometer Readings. For Voltage. (2)	Potentiometer Readings. For Current. (3)	Watts. Dot Product* Col. (2) & Col. (3) × 500. (4)	Watts. Average of Col. (4). (5)	Watts. D.C. Calibration. (6)
100	1·2189 + j0·2457	0·1553 + j0·0447	100·14		
	1·2173 + j0·2460	0·1555 + j0·0447	100·14	100·1	99·7
200	1·2133 + j0·2421	0·3116 + j0·0889	199·80		
	1·2126 + j0·2417	0·3121 + j0·0888	199·96		
	1·2084 + j0·2399	0·3130 + j0·0884	199·72	199·8	199·7
300	1·1910 + j0·2430	0·4700 + j0·1678	300·30		
	1·1784 + j0·2322	0·4742 + j0·1792	300·20		
	1·1790 + j0·2339	0·4735 + j0·1798	300·15	300·2	300·3
400	1·1547 + j0·2231	0·6491 + j0·2262	399·99		
	1·1568 + j0·2233	0·6478 + j0·2258	399·90		
	1·1925 + j0·1536	0·6571 + j0·1067	399·99	400·0	399·5
500	1·1784 + j0·1475	0·8305 + j0·1410	499·73		
	1·1782 + j0·1470	0·8307 + j0·1403	499·68		
	1·1775 + j0·1465	0·8314 + j0·1398	499·73	499·7	500·1
600	1·1542 + j0·1659	1·0115 + j0·1941	599·89		
	1·1535 + j0·1652	1·0118 + j0·1938	599·56		
	1·1499 + j0·1621	1·0159 + j0·1911	599·58	599·6	599·9
700	1·3524 + j0·2560	0·9990 + j0·1860	699·37		
	1·3521 + j0·2582	0·9999 + j0·1858	699·97	699·7	700·1

* Dot or scalar product of $(a + jb)$ and $(c + jd)$ is $ac + bd$.

by two vectors measured upon a co-ordinate system not truly in quadrature, the true expression is

$$\begin{aligned} P &= (a_1 + b_1 \cos\theta)(a_2 + b_2 \cos\theta) + b_1 b_2 \sin^2\theta \\ &= a_1 a_2 + b_1 b_2 + (a_2 b_1 + a_1 b_2) \cos\theta. \end{aligned}$$

The error introduced by neglecting this correction is

$$(a_2 b_1 + a_1 b_2) \cos\theta \text{ watts.}$$

This correction term is really two sine waves in phase with each other. It will be a maximum when the two vectors are both in phase and at 45° to the co-ordinate system of the potentiometer. When both of the vectors are at the a or b axis respectively, or alternatively when the two vectors are equal and symmetrical about either axis the error will be zero. In the former case one of the components of each vector is zero, and in the latter case $a_2 b_1 = - a_1 b_2$.

In the calculation of the reactive volt-amperes the true value will be

$$\begin{aligned} P_j &= (a_1 + b_1 \cos\theta) b_2 \sin\theta - (a_2 + b_2 \cos\theta) b_1 \sin\theta \\ &= (a_1 b_2 - a_2 b_1) \sin\theta. \end{aligned}$$

Since $\sin\theta$ is nearly unity, it will be evident that the correction to be applied is negligible. If the angle θ differs from 90° by 15′ the calculated value differs from the true value by only 1 part in 1,000,000, whilst an error in the quadrature of over $1\frac{1}{2}$° would be necessary to produce an error of calculation of 1 part in 2,000, and thus it may be said that the measurement of stored energy is unaffected by errors in the quadrature of the co-ordinate system and is always $P_j = a_1 b_2 - a_2 b_1$ reactive volt-amperes.

An error in the setting of the co-ordinate system would be just as serious in its effects upon the power measurements as an equal error in the knowledge of the phase angle in the polar type of measurement. This error is more easily avoided with the co-ordinate system because the true quadrature between the axes can be very accurately maintained. The error in quadrature should not exceed a few minutes in a properly adjusted co-ordinate potentiometer.

A further error may be introduced by the variations of frequency as mentioned in Chapter VIII. If the frequency of

the supply is 0·2 per cent. higher at the time of measurement than at the time of standardization of the potentiometer the true power will be $a_1a_2 + (1{\cdot}002)^2b_1b_2$ whilst the apparent measured power is $a_1a_2 + b_1b_2$. If the permissible error due to this cause is 1 part in 2,000 it can be shown by an argument similar to that given in Chapter VIII that $\frac{b_1}{a_1} \cdot \frac{b_2}{a_2}$ will be 0·125. To reduce the error to a negligible quantity it is therefore necessary to arrange for one or both of the vectors to be along or very close to the a axis.

The error in the measurement of stored energy in this case is the same as the error in the frequency and cannot be eliminated, for it is impossible to reduce the quadrature components to zero.

It is of interest to note that in the tables of results of the wattmeter calibration given on page 123 the product $\frac{b_1}{a_1} \cdot \frac{b_2}{a_2}$ is of the order of 0·05. In this case the possible error due to a frequency variation of 0·2 per cent. is less than 1 part in 2,000.

In the calibration of an instrument of the type shown by the table on page 123 where mechanical errors are reduced to a negligible quantity, it is necessary to allow the instrument to attain a steady state by leaving it in circuit for about half

TABLE VI.

CALIBRATION OF A WATTMETER.

Manufacturers . .	Weston Electrical Instrument Corporation.
Type	Dynamometer.
Range . . .	0·5 amp. 150 volts (*i.e.*, 750 watts).
Scale	Uniform—diagonally divided to 1 watt.

Instrument Reading. (1)	Potentiometer Readings. For Voltage. (2)	Potentiometer Readings. For Current. (3)	Watts. (4)
400	$1{\cdot}2127 + j0{\cdot}0497$	$0{\cdot}6566 + j0{\cdot}1000$	400·61
	$1{\cdot}2106 + j0{\cdot}0629$	$0{\cdot}6560 + j0{\cdot}1069$	400·44
	$1{\cdot}1936 + j0{\cdot}1539$	$0{\cdot}6565 + j0{\cdot}1068$	400·02
	$1{\cdot}1925 + j0{\cdot}1536$	$0{\cdot}6571 + j0{\cdot}1067$	399·99

Time taken to obtain above figures—approximately 15 mins.

an hour before taking any readings on the potentiometer, the latter having itself been in circuit for a still longer time. In the table given above, the wattmeter indication was adjusted to a scale marking and successive readings of current and voltage were taken over a period of 15–20 minutes. It will be noticed that although the indication of the instrument remained constant the power measured by the potentiometer steadily decreased. The variation is, of course, small but the figures serve to illustrate the necessity for allowing a state of equilibrium to be attained.

Calibration of a Sine Meter.—The method of calibrating a sine meter is identical with that of a wattmeter. The sine meter should read reactive volt-amperes. The true reactive volt-amperes are found from the measured vector values of current and voltage in the same way as the power is found except that the reactive volt-amperes or EI sin φ is given by $P_j = a_1b_2 - a_2b_1$ as explained above.

The Power Factor.—The power factor at which the wattmeter or sine meter is being tested can at once be derived from the ratio of the reactive volt-amperes to the power. That is if the current and voltage are displaced by the angle φ

$$\tan\varphi = P_j/P$$

$$= \frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}$$

$$\cos\phi = \frac{P}{\quad}$$

NOTES AND BIBLIOGRAPHY.

1 Steinmetz, C. P.: "Theory and Calculation of Electric Circuits," pp. 91–110, 1917 (McGraw & Hill).

2 Connelly, D.: "Precision Measurements with an A.C. Potentiometer," *J. Royal Technical College*, 1937, pp. 159–72.

CHAPTER X.

USES OF THE A.C. POTENTIOMETER (*continued*).

The Measurement of Iron Losses.—The measurement of the magnetizing losses in iron can be carried out very easily by means of the a.c. potentiometer, and in America this instrument is specified for this purpose.[1] The usual method of testing is to measure the exciting current by the voltage drop in a non-inductive resistor in series with the primary circuit, and the induced secondary voltage upon a suitable search coil wound upon the iron sample. As long as the magnetization of the iron is occurring over a reasonably straight portion of the BH curve, the induced secondary voltage with a sine wave of exciting current and vice versa will be reasonably free from harmonics, and a fairly accurate balance can be obtained in the measurement of current and voltage by means of the a.c. potentiometer. The very selective nature of the vibration galvanometer used for balancing rejects the harmonics which are not balanced so that the balance of the fundamental is obtained with sufficient precision for most iron-testing purposes. If the induced voltage is of the order of 1 volt, then a balance to the nearest millivolt can usually be obtained, which is perfectly satisfactory for a great number of tests. When it is necessary to make measurements at high flux densities, the harmonics introduced into the induced voltage or exciting current, according to whether sine voltage or sine current is used, become so large that they form an appreciable part of the whole voltage measured. In this case special methods of flux measurement are required; this matter will be dealt with in further detail in a later section.

From the measurement of exciting current and induced voltage it is possible to calculate the power loss in the iron due to hysteresis and eddy currents, the energy stored, which is as characteristic of the quality of the iron as the power loss,

and the permeability, all under the condition of a.c. excitation which is the normal condition in the iron circuits.

A ring sample of uniform section of the material is preferable, and upon this, a winding of fine wire uniformly spaced close to the surface should be wound but not exercising any mechanical constraint on the iron. This is to form the secondary winding. The turns should be chosen to suit the maximum flux density to which the sample is to be subjected. For example, if the cross-sectional area is 5 sq. cm. and a flux density of 10,000 lines per sq. cm. is to be measured, the total flux will be 50,000 lines. The maximum voltage should not exceed say 1·50 volts, thus the number of turns derived from the well-known relationship $E = \sqrt{2}\pi N_2 f\Phi' \times 10^{-8}$ volts will at 50 cycles give a value of N_2 not more than 13·5 turns, say 12 turns. Over this winding the exciting primary turns should be wound of sufficient section to carry the desired exciting current. If a value of H' equal to say 3 oersteds is required and the mean length l of the ring is 30 cm.

$$H' = \frac{4\pi}{10}\frac{IN_1}{l}.$$

That is
$$3 = \frac{1{\cdot}256 IN_1}{l}.$$

If I is not to exceed say 1 ampere r.m.s. value

$$N_1 = \frac{3 \times 30}{1{\cdot}256} = \text{say } 70 \text{ turns.}$$

No. 17 Copper would be a suitable size of wire for this exciting winding. A non-inductive resistor of say 1 ohm should be connected in series with the exciting winding, and this circuit supplied through a series resistor or conductance box, or variable choke from the same source of supply as the a.c. potentiometer. Current would then be passed through the exciting winding and the volt drop

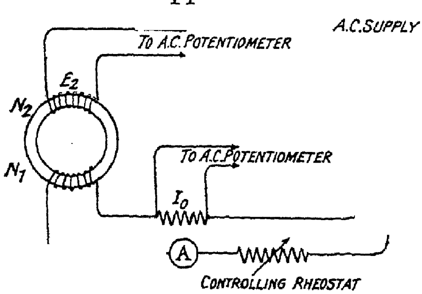


FIG. 72.—Iron testing by means of the a.c. potentiometer.

on the resistance and the induced voltage in the secondary winding measured by means of the a.c. potentiometer. Fig. 72 shows the circuit. Calling the exciting current $I_0 = a_0 + jb_0$ and the secondary induced voltage $E_2 = a_2 + jb_2$, the power loss in the circuit will at once be given by

$$P_0 = (a_0 a_2 + b_0 b_2)\frac{N_1}{N_2} \text{ watts.}$$

The value of flux density can be determined from E_2 and will be

$$B' = \frac{\sqrt{a_2^2 + b_2^2}}{\sqrt{2}\pi f N_2 S} \cdot 10^8 \text{ gauss.}$$

Substituting values gives

$$B' = 7500\sqrt{a_2^2 + b_2^2}.$$

The value of P_0 can be plotted against the corresponding value of B' in the usual way.

Fig. 73 shows the value of the power loss in a sample of iron stampings determined in this way.

When the iron is magnetized, energy is stored in the magnetic field, so that the total energy of magnetization consists

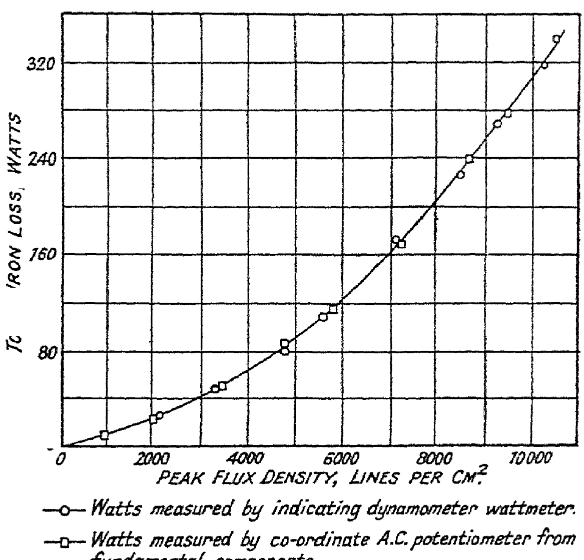


FIG. 73.—Curve showing the power loss in a sample of iron.

of the part transformed into heat, or the power loss and the other part stored in the magnetic field. These two components can be expressed symbolically as real power and imaginary power exactly analogous to the symbolic representation of the real and imaginary components of the complex quantity representing current or voltage.

The energy stored in the magnetic field will be given by

$$P_j = (a_2 b_0 - a_0 b_2)\frac{N_1}{N_2} \text{ reactive volt-amperes.}$$

This can be converted into joules by dividing by $4\pi f$, as already shown, so that the energy stored in joules will be

$$w_S = (a_2 b_0 - a_0 b_2)\frac{N_1}{4\pi f N_2} \text{ joules.}$$

This quantity is just as characteristic of the iron as the power loss, and can be related to the permeability, because the energy stored depends upon the flux, the current and the turns, the relation between which is also expressed by the permeability of the iron.

Complex Permeability.[2-3]—The permeability of the iron when excited by alternating current cannot be expressed by a single numerical quantity because the fundamental sinusoidal component of the flux is not in phase with the current which produces it. In order to express this phase displacement between magnetizing force and the resulting induction it is necessary for permeability to become a complex quantity of the form $\mu = a + jb$. The flux B produced by a magnetizing force H then becomes

$$B = \mu H = (a + jb)H.$$

This can be shown in the following manner:

Let $B' \equiv$ maximum value of flux density
$H' \equiv$ maximum magnetizing force
$S \equiv$ cross-section of the iron in sq. cm.
$f \equiv$ frequency
$l \equiv$ length of magnetic circuit in cms.
$N_1 \equiv$ exciting turns
$N_2 \equiv$ secondary turns
$I_0 \equiv$ exciting current $= a_0 + jb_0$
$E_2 \equiv$ induced secondary voltage $a_2 + jb_2$.

Then $$B' = \frac{jE_2}{\sqrt{2}\pi f N_2 S}10^8$$

and $$H' = \frac{\sqrt{2}\, 4\pi N_1}{10l}I_0$$

from which $$\mu = \frac{B'}{H'} = \frac{10l}{8\pi^2 S f N_1 N_2}10^8 j\frac{E_2}{I_0}.$$

Substituting the measured values of E_2 and I_0 and putting

$$\frac{10l}{8\pi^2 S f N_1 N_2} = K$$

$$\begin{aligned}\mu &= K\frac{jE_2}{I_0} = K\frac{-b_2 + ja_2}{a_0 + jb_0}\\ &= K\left[\frac{a_2 b_0 - a_0 b_2}{a_0^2 + b_0^2} + j\frac{a_0 a_2 + b_0 b_2}{a_0^2 + b_0^2}\right]\\ &= a + jb.\end{aligned}$$

It will be seen that the two components a and b of the complex permeability are proportional to the stored energy and the power loss in the sample. The component a gives the energy storing property of the iron while the component b represents the energy absorbing (*i.e.*, transforming to heat) property of the iron.

That is, $$a = KP_j/I_0^2$$

and $$b = KP_0/I_0^2.$$

The ratio of b to a gives the tangent of the phase angle between the exciting current and the flux it produces.

When a sine wave of current is used to excite a sample of iron under test, the magnetizing force H will be sinusoidal, but the resulting value of flux will be non-sinusoidal, or will contain harmonics if the relation between B and H is non-linear. This relationship is given by the hysteresis loop for the sample.

The a.c. potentiometer measures only the sinusoidal components of fundamental frequency and ignores the harmonics, so that in measuring the induced voltage E_2 only that part which is proportional to the rate of change of the fundamental sinusoidal component of the flux is measured.

If the magnetizing current is a sine wave, the power loss in the iron measured by the a.c. potentiometer will be correct

because the harmonics in the induced voltage when multiplied by the current, produce an average value which is zero. That is, the power is given by the product of the quantities of the same frequencies only. The product of quantities of different frequencies gives a periodic function, the average value of which is zero.

Losses in the iron due to harmonic frequency components of the flux are compensated by an equivalent increase in the power loss at the fundamental frequency. That this is experimentally true is shown by Fig. 73, which shows the comparison of power measured by the wattmeter and by the a.c. potentiometer.

The energy loss during the cycle of magnetization is given by the area of the hysteresis loop plus the eddy current loss. The eddy current loss will be proportional to the square of the flux and therefore follows another loop which is proportional to the square of the actual flux cycle loop. It can be replaced for practical purposes by an ellipse, the area of which is equal to the eddy current energy loss per cycle.

When the total iron loss is measured by means of the a.c. potentiometer the two components of the complex permeability define an ellipse whose area is the area of the hysteresis loop plus the equivalent eddy current ellipse. This is the loss ellipse of the iron. The actual co-ordinates of the loss ellipse may be expressed in volts and amperes or in B and H, in which case the units are the same as in the usual hysteresis loop. The energy is then given by

$$\frac{B'H'}{4\pi} \text{ ergs per cycle}$$

or

$$\frac{B'H'}{4\pi}.f \text{ ergs per second}$$

or

$$\frac{B'H'}{4\pi}.f10^{-7} \text{ watts.}$$

The equation to the ellipse will be given by

$$B = aH \pm b\sqrt{H'^2 - H^2}$$

where B is the value on the ellipse corresponding to the value of H, a and b the coefficients of the complex permeability

measured as described above, and H′ is the value of maximum magnetizing force given by the same measurements.

There is therefore a separate ellipse corresponding to every value of B′ to which the sample is subjected. Each point of the curve in Fig. 73 represents the area of one of these ellipses corresponding to the particular value of B′. It is seldom necessary to make an actual plot of the ellipses, since the watt-loss curve is more relevant, but its significance should be understood.

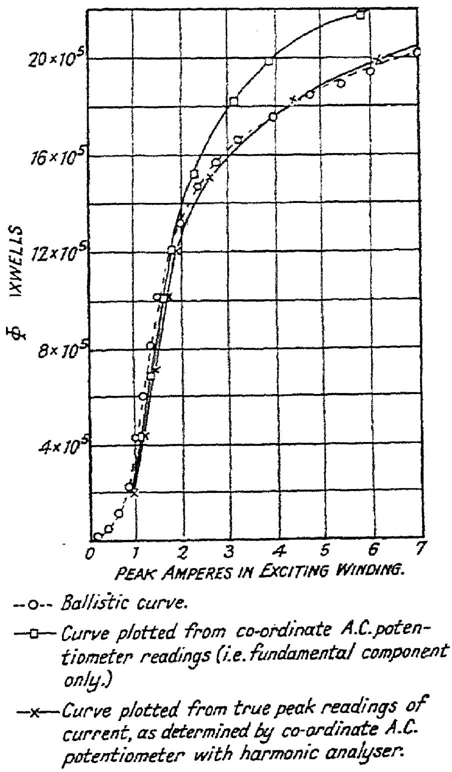


Fig. 74.—Magnetization curves for a sample of iron.

Although the fundamental component of the flux is displaced in phase with respect to the current producing it, and has its maximum some time after the magnetizing force, the actual maximum value of the flux must coincide with the maximum value of the magnetizing force. The true maximum value is the point of the hysteresis loop and not the maximum value of the sinusoidal ellipse.

The value of B′ given by the a.c. potentiometer measurement is therefore not the true maximum value but will be invariably greater at the higher inductions than the value corresponding to the hysteresis loop obtained by d.c. tests for the same value of maximum magnetizing force.

The true maximum value of B′ can be obtained by the use of an a.c. and d.c. potentiometer and the harmonic analyser

(see page 139). When this is done the values of B′ corresponding with H will follow the same curve as is obtained by means of the d.c. test. These points are shown in Fig. 74.

The equivalent ellipse is higher than the true loop although the area is the same. This is shown for one value of induction in Fig. 75.

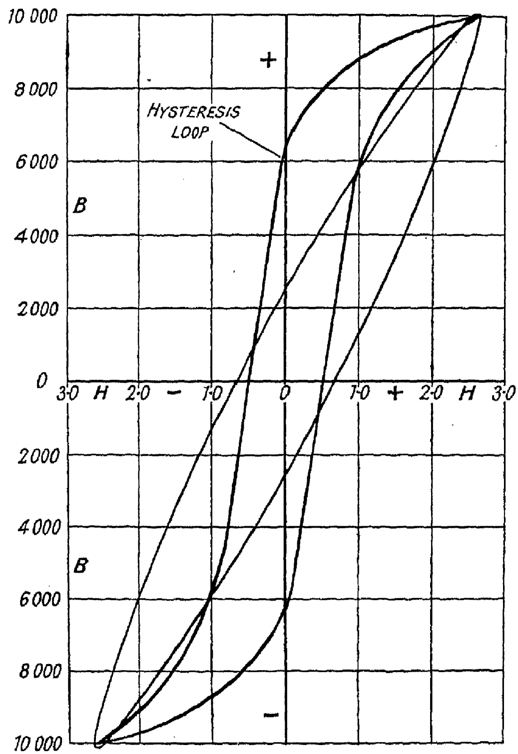


FIG. 75.—The hysteresis loop and ellipse.

It should be clearly understood that the flux in the iron does not actually follow the ellipse, but conforms much more closely to the hysteresis loop, traversing this each cycle. The ellipse is what is implied by the measurements made with a.c., and is the only interpretation which can be put upon such measurements at one frequency.

It will be obvious therefore from Fig. 75 that the a.c. measure-

ments of flux and magnetizing force give a representation which is very different from the actual state of affairs although this representation can be used with confidence in predetermining magnetic effects in iron. It must never be forgotten that like most other mathematical representations of physical problems, it is only a convenient symbolism to express experimental results.

For most design purposes the a.c. potentiometer values are more convenient than the true hysteresis loop derived from step by step d.c. measurements, because they give the actual behaviour of the iron under conditions of a.c. excitation. For transformer design and the comparison of specimens the a.c. values are certainly the more convenient.

The departure between the loop and the ellipse is greater the higher the induction. As in the case of the power, the harmonics do not contribute to the mean stored energy or reactive volt-amperes if either the exciting current or the flux is a sine wave, and the value given by the a.c. potentiometer measurement is correct. The harmonics give values oscillating about zero so that the instantaneous value of the stored energy does contain the harmonics, just as the instantaneous power does. Both consist of sine waves of double the frequency of the voltage or current, rising and falling from zero to a maximum, the harmonics being oscillations of higher frequency superimposed, but symmetrical about the zero, and therefore not affecting the mean value.

If the iron is excited by a sine wave of voltage, the flux will be very nearly a sine wave. In this case exactly the same methods of test apply and the above reasoning applies equally well. The induced voltage will now be a sine wave and the exciting current, and therefore the magnetizing force, will be distorted.

In testing iron samples, methods are usually employed to produce either a sine wave of current or alternatively a sine wave of flux. A large series resistance or impedance in the exciting circuit gives an exciting current of sensibly the same wave form as the supply voltage. With a very low impedance in series with the exciting circuit, the voltage approximates very closely to the supply voltage. If this is sinusoidal, the

flux which is the integral function of the reactive voltage will also be sinusoidal, since the integral of a sine wave is a cosine wave.

Testing with a sine wave of voltage is now generally preferred because it corresponds more nearly with operating conditions. There are, however, practical difficulties in reducing the circuit impedance sufficiently to maintain a sine voltage at high excitations.

When both current and voltage are non-sinusoidal, the power and stored energy are products of the harmonics as well as of the fundamentals. When the harmonics are large in both the current and the voltage, the simple a.c. potentiometer will not measure the power loss or the stored energy.

The presence of harmonics in a voltage wave makes it difficult to measure by means of the a.c. potentiometer, because the potentiometer can only balance out the fundamental frequency, leaving a free voltage equal to the sum of all the harmonics, acting in the galvanometer circuit. In practice, their effect makes it impossible to get a clean balance or sharp image of the vibration galvanometer spot. Unless the harmonics are very strong a fairly good minimum balance for the fundamental can be obtained, and the galvanometer spot usually exhibits a curious kind of double or multiple image which moves from side to side and re-forms into different figures as the balance point, or minimum balance of the potentiometer, is varied. If this complex image of the galvanometer is analysed by a rotating mirror, it will be found that the spot is vibrating in a forced complex wave corresponding in some degree to the harmonics, but conditioned by the natural period of the suspension vibrating in various modes. The multiple image will be much reduced in the case of an efficient moving-coil type of vibration galvanometer by heavily shunting the coil by a resistance, but this will, of course, reduce the sensitivity corresponding to the fundamental balance. In many cases, however there is a decided gain in the precision with which the minimum balance point can be determined, by using a shunt.

Measurements on Circuits containing Harmonics.— It has been pointed out already that the a.c. potentiometer is essentially an instrument which measures the fundamental

component of the frequency at which the test circuit and the potentiometer are excited. The presence of harmonics in either of these circuits leads to difficulties, both in obtaining a precise balance and in the interpretation of results, so that for general purposes it is advisable to use a source of supply which is extremely pure and free from harmonics. In the case of iron testing this becomes impossible, because if a sample of iron is excited with a sine wave of a.c. the flux induced in the iron will not be sinusoidal owing to the curvature of the relationship between the induction and the magnetizing force. The e.m.f. induced in the winding is proportional to the rate of change of the flux, and because the flux is non-sinusoidal the rate of change will be non-sinusoidal. Since the flux in the iron is determined from the voltage induced in a secondary winding, and this secondary voltage will be non-sinusoidal if the iron is excited with a sine wave of current, it is essential to have some means of measuring this non-sinusoidal voltage in order to determine the magnetic properties of the iron. At high values of flux density the harmonics introduced into the secondary voltage become so large that they form an appreciable part of the whole flux, so that a flux value derived from the fundamental component alone would be seriously different from the total flux acting in the iron.

To determine the properties of the iron with greater accuracy, it is therefore necessary to adopt some method by means of which the induced voltage wave-form may be resolved into its component frequencies. For this purpose a synchronous contact-maker is used, operating in the galvanometer circuit of the potentiometer.[4]

This contact-maker is operated by a synchronous motor driven from the same source of a.c. supply that is used for exciting the iron. The synchronous motor should be arranged to operate a contact, the phase and duration of which can be adjusted. It is further necessary that the contact-maker should be of such a form that the number of contacts per cycle of the synchronous motor can be any integral value, so that three, five or seven contacts per cycle can be made, the duration of which can be varied and the phase of which can also be altered with respect to the supply. By means of this device any com-

plex a.c. wave can be analysed. The method of procedure is as follows: the synchronous contact-maker is introduced in series with the galvanometer used in conjunction with the potentiometer. It is well known that the contour of an a.c. wave can be copied by means of a single contact-maker, the phase of which can be successively moved so that a measurement of the instantaneous value of the wave can be made at successive points along each cycle. This is the Joubert contact-maker,[5] and the simple circuit using a d.c. potentiometer as a means of balancing the instantaneous value is shown in Fig. 76. With the Joubert contact-maker only one contact is made per cycle. If now the number of contacts is increased to three per cycle, the contacts being spaced 120 electrical degrees apart, it is easy to show that the average value of the

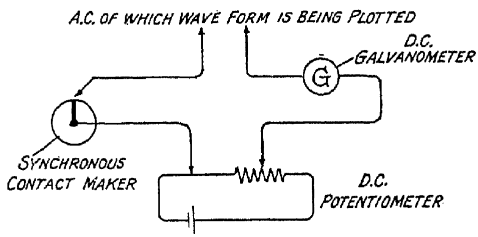


FIG. 76.—Wave-form analysis by the Joubert contact-maker.

three ordinates of any wave will be zero except those having a period corresponding to the number of contacts per cycle or multiples therof. For example, if three contacts are made on the fundamental frequency, the mean value of the three ordinates equally spaced throughout the cycle will be zero, no matter at what phase they occur, but so far as the third harmonic is concerned the three contacts would always occur at the same point in the wave and the voltage at that instant will be the amplitude of the third harmonic at that instant and this can, therefore, be measured upon the d.c. potentiometer in exactly the same way as with the simple Joubert contact-maker. The same applies to whatever number of contacts per cycle are used, so that the higher harmonics can be analysed by this multiple contact up to the limit when the accuracy of contacting fails. The upper limit is usually about the eleventh

or thirteenth harmonic on a 50-cycle supply, at which the equality of the contact becomes insufficiently exact to give the mean reading. Harmonics which are multiples of the number of contacts can be eliminated by varying the duration of the contact time.[6-7]

For examination of the harmonics in the iron-testing circuit the synchronous contact together with the d.c. potentiometer are put in series with the a.c. potentiometer and galvanometer as shown in Fig. 77. The fundamental component of the a.c. voltage is balanced as nearly as possible by means of the a.c. potentiometer in the normal way with the synchronous contact-maker short-circuited. Having obtained the a.c. balance in

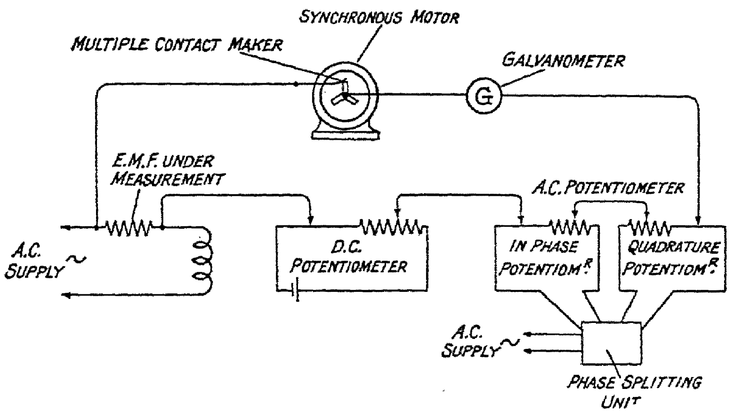


FIG. 77.—Use of a.c. and d.c. potentiometers in harmonic analysis.

this way, the synchronous contact-maker is then started and with the contact-maker making three contacts per cycle the third harmonic is balanced out by means of the d.c. potentiometer and the contour of the harmonic plotted by altering the phase of the contact in exactly the same way as when using a Joubert contact-maker. The process is repeated for the successive harmonics. In practice it is only necessary to find the zero point and the maximum amplitude of the harmonic and there is no need to plot successive points. These two points are found very readily by setting the d.c. potentiometer to zero and turning the phase of the contact-maker until the d.c. galvanometer is balanced. By turning the phase once more to

the appropriate phase angle corresponding to the quarter period of the harmonic under investigation and adjusting the d.c. potentiometer to balance, the magnitude at its greatest height is measured. This contact-maker is of the greatest utility in iron testing as it is suitable for a number of uses apart from those just described. By arranging for the contact to close for a half-cycle only the device then acts as a half-wave rectifier and is one of the most convenient means of determining the maximum flux in an iron circuit and can be used in a similar way for the determination of incremental permeability [8] when alternating and direct fluxes are superimposed in the iron. When used as a rectifier, the duration of the contact time must be set to correspond to the half-period of the cycle and the phase adjusted so that the contact is closed throughout the positive half of the wave. If, now, the voltage is measured upon the d.c. potentiometer under these conditions by balancing the rectified voltage, the potentiometer will give an average value of the voltage throughout that period, irrespective of the wave-form. Since the induced voltage e is proportional to $\frac{d\Phi}{dt}$ it is obvious that the integral of edt is proportional to the integral of $d\Phi$. The integral of edt between the limits of 0 and $\pi/2$ will be the average value of the secondary voltage. It is clear, therefore, that the average value of the secondary voltage is a direct measurement of the total flux. In this way the synchronous rectifier in conjunction with the d.c. potentiometer affords one of the simplest methods of measuring flux in an a.c. circuit. If the flux cycle is unsymmetrical, as in the case of superimposed d.c., then it is only necessary to alter the duration of the contact to agree with the unequal portions of the wave. This condition can be found experimentally by successive adjustments of the contact time. The two unequal half-cycles can then be measured independently by altering the phase and duration of the contact appropriately, and in this way, the flux for each unequal half-cycle can be determined.

In practice when using the synchronous contact-maker for flux measurements, it is useful to have a mutual inductance in the primary circuit as a convenient means of calibration. The flux corresponding to the mutual inductance will by definition

be equal to $\varphi = \frac{Mi}{N}$ where M is the mutual inductance in henries, i the primary current in amperes and N the number of turns in the secondary winding. It is therefore possible to get a direct reading of a known flux upon the d.c. potentiometer using the synchronous rectifier, for comparison with the iron flux.

The d.c. potentiometer shown in Fig. 77 is sometimes replaced by a moving-coil d.c. voltmeter. This instrument will indicate the average secondary voltage in the rectified circuit, but it must be used with caution. It will not give the same reading as the potentiometer even when it has a high resistance. The reason is easy to see if the current in the circuit is con-

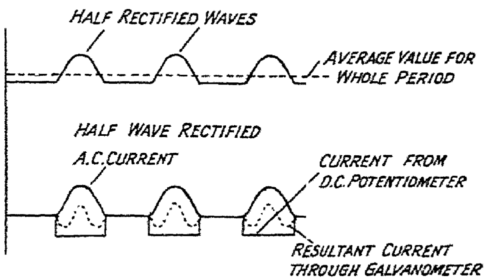


FIG. 78.—Currents in circuit of Fig. 77, using (*a*) voltmeter, (*b*) d.c. potentiometer.

sidered; a moving-coil voltmeter is really a milliammeter and its indication will be the average value of the current flowing due to the secondary voltage rectified by the half-wave synchronous contact-maker. This current will consist of a series of half-period waves, which the moving coil will integrate as indicated in Fig. 78, integrating over the whole cycle. So long as the power taken is negligible compared with the exciting power, the voltmeter used in this way will give a value of about half the average voltage as a measure of the flux, because only half the wave is rectified.

When the potentiometer is used, the galvanometer, which is also a moving-coil instrument, is called upon to balance the current sent round the circuit by the potentiometer against the current sent round by the rectified half-wave of the a.c. secon-

dary voltage. Both voltages are acting in the same circuit and the impedance for both will be very nearly the same in most cases. The galvanometer will appear balanced when the average current through it is zero. This will occur when the average value of the current from the potentiometer is equal to the average current from the rectified a.c. voltage. This condition is indicated in the lower part of Fig. 78. There will be a current through the galvanometer when it reads zero, of zero average value. The potentiometer reading will be equal to the average value of the voltage, not half the value as in the case of the voltmeter, because the circuit of the potentiometer is only active during the time the contact is closed. The galvanometer should be one of reasonably long period, not affected by the triple frequency current passing in its coils, in order that it may give steady readings. This case is interesting because it is a potentiometer method of considerable practical importance, in which the solution of the conditions of balance must be dealt with in terms of the pulses of current flowing under the two voltages acting, in order to understand what takes place. The potentiometer gives the average value of the rectified pulse while the voltmeter integrates the whole rectified cycle, and therefore gives a lower value which needs careful interpretation. It is interesting to note that if the resistance of the voltmeter is increased to infinity, the voltmeter and potentiometer will read the same, because there is no loss of current when the rectifier contact is open. This is approximately the case with a valve voltmeter, but the valve voltmeter is not an integrating instrument but more nearly a peak instrument, so that its use for this purpose needs careful qualification.

Predetermination of the Ratio and Phase Errors of Current Transformers.—The errors of current transformers can be easily computed from measurements made upon the iron core at flux densities corresponding to various loads.

Since the errors are due to the energy loss and the storage of energy in the iron circuit, the measurements necessary for the determination of these quantities are sufficient also for the predetermination of the current transformer ratio and phase angle errors.

In practice it may be necessary to wind a special coil upon

the core for test purposes, taking care not to constrain the iron; or alternatively the main winding can be used if suitable shunts are available for measuring the exciting current. It is only necessary to measure the exciting current required to produce the various flux densities in the core and the corresponding induced voltage. Suppose the secondary to have 100 turns and the ammeter which is to be used with this winding to require 0·5 volt at full load of 5 amperes. A temporary winding of, say, 50 turns might be wound upon the iron core and used for exciting purposes, this winding being fed from the supply through a suitable controlling rheostat as shown in Fig. 72 and a non-inductive resistance of say 0·1 ohm inserted in series upon which the volt drop, and therefore the current, could be measured by means of the a.c. potentiometer. At the same time the voltage induced in the open-circuited secondary winding should be measured. Calling these two values I_0 and E_2 respectively, the power loss and reactive volt-amperes will be given by

$$P = R(a_0a_2 + b_0b_2) \text{ watts}$$

and

$$P_j = R(a_0b_2 - a_2b_0) \text{ reactive volt-amperes.}$$

The term R is the turns ratio necessary to reduce the induced voltage E_2 to the same scale as the exciting current I_0.

If I_1 is the primary current, I_2 the secondary current and I_0 the exciting current, measured on the secondary side, the total primary current

$$I_1 = RI_0 + RI_2$$

where R is the turns ratio or N_2/N_1 since I_1 will be larger than I_2

$$\therefore \frac{I_1}{I_2} = R\left(1 + \frac{I_0}{I_2}\right).$$

On non-inductive secondary load it is also easy to show[9] that the above relationship can be separated into the ratio error and the phase angle error since $I_2 = P_2/E_2$ and the exciting current in phase with the load current will be RP_0/E_2

$$I_1 = R(P_2/E_2 + P_0/E_2).$$

$$\therefore I_1/I_2 = R(1 + P_0/P_1).$$

$$\text{That is the ratio error} = R \cdot \frac{P_0}{P_2}$$

where R is the turns ratio and P_2 the power taken by the secondary load and P_0 the iron loss at that particular power output (as measured on the secondary side). Similarly the phase angle error will be given by the component of exciting current in quadrature with the load current. This is RP_j/E_2. The phase difference produced by this will be

$$\tan \psi = \frac{P_j}{P_2}$$

where P_j is the reactive volt-amperes of excitation (or stored energy). That is, the smaller the exciting power and reactive volt-amperes in relation to the load power, the more perfect the transformer.

This method of determining the ratio and phase errors of current transformers is capable of a very high degree of precision. It is interesting to note that the energy loss transformed into heat governs the ratio difference, and the storage of energy or reactive volt-amperes governs the phase difference between input and output. This is true of any network.

From the actual vector measurements of I_0 and E_2

when $\qquad I_0 = a_0 + jb_0$

and $\qquad E_2 = a_2 + jb_2$

the current ratio factor K_C at any secondary loading of the transformer P_2 watts will be

$$K_C = \frac{N_2}{N_1}\left(1 + a_0a_2 + b_0b_2\right.$$

where N_2 and N_1 are the secondary and primary turns respectively used in the measurement of I_0 and E_2. (That is I_0 is measured on the N_2 side and E_2 on the N_1 side.) These need not be the actual windings used on load so long as the value of E_2 is that appropriate to the flux density which would be acting in the core for the load P_2, but the value of P_2 must include the power loss in the secondary winding. More strictly, the leakage reactance and effective resistance of the secondary circuit should be included in the load, but the leakage reactance of a good current transformer wound toroidally on a closed core is very small, so that only the secondary resistance need be taken.

Similarly the phase angle difference between primary and secondary currents will be given by

$$\psi = \tan^{-1}\frac{a_0b_2 - a_2b_0}{P_2}.$$

The sign of the numerator (which is P_j) can always be taken as positive in a simple circuit which does not contain both inductance and capacitance although the actual measured signs may give a negative value. On non-inductive load the secondary current will always lag behind the primary current reversed. That is, the true angle between them will always be less than 180°.

The behaviour of the transformer on inductive load can always be derived from these measurements by considering the effect of shifting the phase of the load current before adding the exciting current. On inductive load, the load current will tend to come more into phase with the exciting current, and therefore the ratio error will increase, but the phase angle error decreases, until the load current has the same phase angle as the exciting current. This has the phase angle of

$$\phi = \tan^{-1}\frac{P_j}{P_0}$$

or

$$\tan\phi = \frac{a_0b_2 - a_2b_0}{a_0a_2 + b_0b_2}.$$

On capacitance load the ratio error of the transformer would decrease and the phase angle error increase until the secondary load current had a phase displacement leading the secondary voltage by the angle $90 - \varphi$ when there would be no ratio error but a maximum phase error.

The maximum possible ratio error on inductive load can be derived from

$$K_C = \frac{N_2}{N_1}\left(1 + \frac{\sqrt{P_0^2 + P_j}}{P_2}\right.$$

and the maximum possible phase error on capacitive load can be

$$\psi = \tan^{-1}\frac{\sqrt{P_0^2 + P_j^2}}{P_2}$$

the expression $\sqrt{P_0^2 + P_j^2}$ is the numerical value of the exciting volt-amperes.

This method of test is frequently used in the design and manufacture of current transformers for the predetermination of the errors.[9-10] The suitability of a particular iron core can be found by winding two temporary windings, and determining the losses at various flux densities. The final design then follows, the secondary turns being made sufficiently large so that the flux density in the core at the highest secondary voltage (or largest current) will be low enough for the errors due to the iron losses at that flux density to be within the desired values.

The secondary voltage will depend upon the secondary burden including the secondary leakage reactance and resistance, so that the greater the impedance of the secondary burden the higher the flux density and the larger the iron losses will be for the same current. The primary turns follow simply as a ratio of the desired secondary current to the primary current. In low precision transformers using ordinary silicon transformer steels the ratio of the turns is not necessarily numerically equal to the nominal current ratios, but extra secondary turns are allowed to compensate for the large iron losses. In precision transformers using nickel-iron alloy cores, the turns ratio usually agrees with the nominal current ratios because the losses are so very small. Unless the core loss component of the exciting current is a larger percentage of the load current than one secondary turn is of the total secondary turns, turns compensation for the iron losses cannot improve the current ratio of the transformers. If the secondary circuit is open-circuited while the primary current is maintained at full load, the secondary load impedance increases infinitely. The flux density in the core will rise to saturation value since the whole of the primary load current is available for exciting the core. This condition gives rise to excessive secondary voltage which may break down the insulation. This high voltage is produced by the sudden reversal of the saturated flux in the core, at the zero point of each current wave, and corresponds to a very rapid rate of change of flux, thus inducing a very high voltage peak. The saturation of the iron core may

alter its magnetic properties, depending upon the magnetic condition in which it is left, but the core can be normalized by demagnetization by flux reversals of decreasing amplitude, starting at saturation.

There is no danger in open-circuiting the secondary winding during magnetic tests if the primary current is limited to the exciting value, that is to say if the open circuit secondary voltage is not allowed to rise much above its maximum working value.

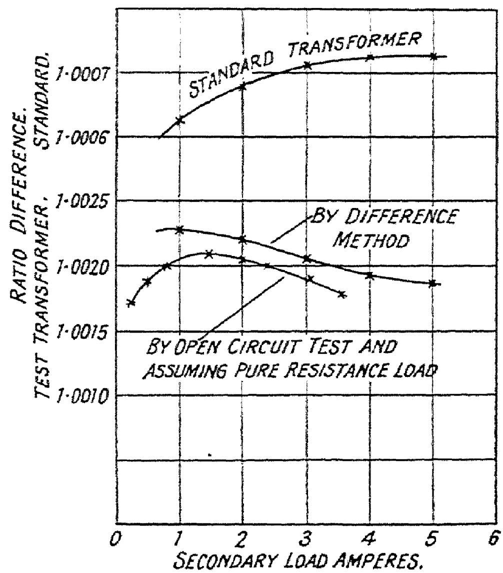


FIG. 79.—Current ratio factor of current transformers.

In Tables VII, VIII and IX are given the results of measurements made by means of the a.c. potentiometer to determine the ratio and phase angle errors of a current transformer.[11] The errors so determined are plotted in Figs. 79 and 80.

The same transformer was then tested by comparison against a standard current transformer. For this method of testing the two primaries are put in series, and the two secondaries are put in series through their proper burdens. The arrange-

TABLE VII.

OPEN CIRCUIT METHOD.

E_I Current.	E_S Volts (Reversed).	E_S Magnitude.	$I = \frac{E_S}{0\cdot482}$.	Ratio. $\frac{K_C}{N_2/N_1}$.	Angle. Minutes.
$0\cdot2200 - j0\cdot0130$ $0\cdot2202 - j0\cdot0130_5$	$0\cdot0381 - j0\cdot1131$ $0\cdot0380 - j0\cdot1130_5$	0·1193	0·248	1·0017	14·2
$0\cdot4105 - j0\cdot0285$ $0\cdot4108 - j0\cdot0287$	$0\cdot1149 - j0\cdot2318$ $0\cdot1149 - j0\cdot2318$	0·2587	0·537	$1\cdot0019_5$	11·7
$0\cdot5201 - j0\cdot0368$	$0\cdot1781 - j0\cdot3043$	0·3526	0·732	1·0020	10·1
$0\cdot8788 - j0\cdot0528$ $0\cdot8787 - j0\cdot0530$	$0\cdot4858 - j0\cdot5372$* $0\cdot4858 - j0\cdot5378$	0·7243	1·5	1·0021	7·0
$1\cdot0946 - j0\cdot0759$ $1\cdot0944 - j0\cdot0750$	$0\cdot7325 - j0\cdot6828$ $0\cdot7325 - j0\cdot6832$	1·0014	2·08	$1\cdot0020_8$	5·7
$1\cdot2512 - j0\cdot0860$ $1\cdot2518 - j0\cdot0862$	$0\cdot9539 - j0\cdot7798$ $0\cdot9529 - j0\cdot7798$	1·2321	2·56	1·0020	4·9
$1\cdot3959 - j0\cdot0951$ $1\cdot3957 - j0\cdot0940$	$1\cdot1760 - j0\cdot8659$ $1\cdot1808 - j0\cdot8678$	1·4604	3·03	$1\cdot0019_5$	4·2
$1\cdot5240 - j0\cdot1021$ $1\cdot5246 - j0\cdot1036$	$1\cdot3988 - j0\cdot9380$ $1\cdot4004 - j0\cdot9408$	1·6842	3·50	$1\cdot0018_9$	3·7

* From this reading onwards the presence of harmonics masked the balance point, necessitating reduction of galvanometer sensitivity. The effect increased with increasing values of E_s. The load current is calculated on the assumption that the circuit resistance of the secondary will be 0·482 ohm. This is made up of the instrument resistance of 0·3 ohm and the resistance of the winding of 0·182 ohm, neglecting the leakage reactance of the secondary circuit.

ment of the circuit is shown in Fig. 81. The a.c. potentiometer is used to measure the difference current between the two transformers by the volt drop in the cross-resistance. The measured values are given below and the errors determined therefrom are plotted in Figs. 79 and 80. These errors are corrected for the known errors of the standard transformer. This method of test is a very useful one, but for the highest pre-

TABLE VIII.

DIFFERENCE CURRENT METHOD.

Ammeter Amperes.	Difference Circuit Volt. E_D	Shunt Resistance Volts. E_S	Ratio Difference $\frac{\text{std. sec.}}{\text{test sec.}}$	Difference Angle. Minutes.
1	$-0{\cdot}00181 + j0{\cdot}00089$	$0{\cdot}1958 + j0{\cdot}0265$	1·00169	0·2
	$-0{\cdot}00180 + j0{\cdot}00090$	$0{\cdot}1958 + j0{\cdot}0266$		
2	$-0{\cdot}00316 + j0{\cdot}00058$	$0{\cdot}3923 + j0{\cdot}0629$	1·00153	0·4
	$-0{\cdot}00321 + j0{\cdot}00058$	$0{\cdot}3927 + j0{\cdot}0629$		
3	$-0{\cdot}00410 + j0{\cdot}00028$	$0{\cdot}5897 + j0{\cdot}0722$	1·00136	0·9
	$-0{\cdot}00411 + j0{\cdot}00027$	$0{\cdot}5930 + j0{\cdot}0712$		
	$-0{\cdot}00410 + j0{\cdot}00021$	$0{\cdot}5911 + j0{\cdot}0782$		
4	$-0{\cdot}00489 + j0{\cdot}00010$	$0{\cdot}7958 + j0{\cdot}0568$	1·00122	1·9
	$-0{\cdot}00490 + j0{\cdot}00009$	$0{\cdot}7951 + j0{\cdot}0567$		
	$-0{\cdot}00489 + j0{\cdot}00010$	$0{\cdot}7948 + j0{\cdot}0565$		
5	$-0{\cdot}00571 + j0{\cdot}00024$	$0{\cdot}9960 + j0{\cdot}0024$	1·00115	3·9
	$-0{\cdot}00571 + j0{\cdot}00025$	$0{\cdot}9939 + j0{\cdot}0039$		
	$-0{\cdot}00571 + j0{\cdot}00024$	$0{\cdot}9990_5 - j0{\cdot}0024$		
	$-0{\cdot}00569 + j0{\cdot}00021$	$0{\cdot}9961 - j0{\cdot}0015$		

Columns E_D and E_S show the consistency obtained in measurement.
Results have been worked out for only one set.

The errors of the standard transformer used in the above test were as below giving the errors of the test transformer.

TABLE IX.

Load Amperes.	Ratio Error of Standard Transformer.	Phase Angle Error of Standard Transformer.	Ratio Error of Test Transformer.	Phase Angle Error of Test Transformer.
	$1{\cdot}00063_9$	4·1	$1{\cdot}0023_2$	8·0
2	$1{\cdot}00068_9$	3·3	$1{\cdot}0022_2$	5·2
3	$1{\cdot}00070_8$	2·8	$1{\cdot}0020_7$	3·7
4	$1{\cdot}00072_0$	2·45	$1{\cdot}0019_4$	2·8
	$1{\cdot}00072_9$	2·3	$1{\cdot}0018_8$	2·5

cision the arrangement given in Chapter XI is preferable, because only one measurement has to be made, and also the volt drop in the cross-circuit is very small with precision transformers.

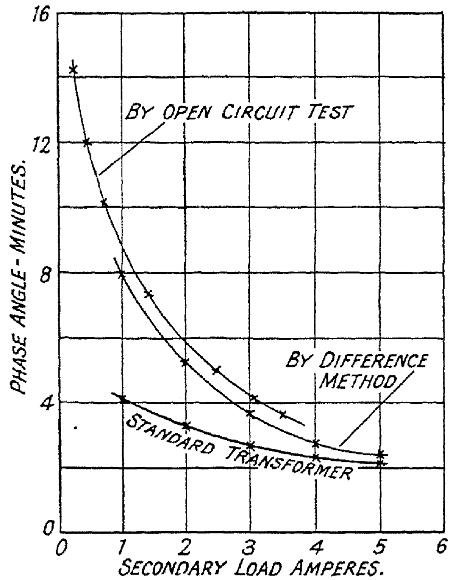


FIG. 80.—Phase angle error of current transformers.

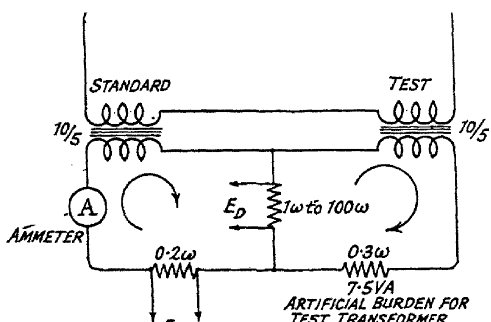


FIG. 81.—Current transformer error determination by difference method.

The small difference between the curves determined by the two methods can be explained by the omission of leakage reactance from the value used for the secondary burden in the

calculation of the ratio and phase angle errors from the open circuit test. The exact load conditions under which the transformer is working are not easy to determine, but this is not of great practical importance as it is more important to be able to assign limits to the errors which will occur when the secondary load does not exceed a certain maximum value. The difference of ratio given by the two methods is only about 1 part in 10,000, except at the lowest values where no normal indicating instrument could be read with any degree of accuracy. Even at $\frac{1}{5}$ load the difference is only about 3 parts in 10,000.

When allowance is made for the secondary reactance in the load, the agreement between the two methods is almost perfect. The secondary load then becomes $0{\cdot}482 + j0{\cdot}0435$ ohm instead of 0·482 ohm. The values calculated from the previous open circuit tests then give the following values:

E_s .	0·1193	0·2587	0·3526	0·7243	1·0014	1·2321	1·4604	1·6842
I . .	0·247	0·535	0·73	1·5	2·07	2·55	3·02	3·48
Ratio	$1{\cdot}0020_5$	$1{\cdot}0022_6$	$1{\cdot}0022_3$	$1{\cdot}0022_8$	$1{\cdot}0022_0$	$1{\cdot}0021_3$	$1{\cdot}0020_6$	$1{\cdot}0019_5$
Angle Minutes	13·6	10·75	9·5	6·4	5·1	4·25	3·65	3·1

The Measurement of Small Phase Angles by the A.C. Potentiometer.—When it is necessary to measure small phase angles, it is always advisable to make the lengths of the voltage vectors embracing the small angle as nearly equal as possible, so that the two voltages and the small angle between them form an isosceles triangle. This triangle will be very nearly a right-angle triangle. The angle between the two sides is then known with high precision if the small difference voltage between them is measured. The measurement of small phase angles occurs in many problems and tests. A simple example is to compare the phase angle of two equal resistors of the four-terminal type, such as are used in measuring current by means of the potentiometer. If the two resistance shunts are placed in parallel (with their current terminals connected) and current passed through them, the volt drop on their potential terminals will be approxi-

mately equal, provided the resistance of the shunts is the same between its current points. This may not necessarily be the case with low-resistance four-terminal resistance standards, but such would generally be the case. If, now, one potential point on one shunt is connected to one potential point on another shunt and the voltage triangle so formed between the potential points is measured upon the a.c. potentiometer, the measurement will give two values of approximately the same length representing the volt drop between the potential points of each shunt and a third value representing the difference voltage between the potential points which are not connected. It is easy to see that the phase angle between these measurements follows from the ratio of the difference voltage to the volt drop between the potential points. If E_1 is the volt drop of one shunt and E_2 the volt drop of the other shunt, and ΔE the difference voltage between them, then vectorially $E_1 - E_2 = \Delta E$ (see Fig. 82). If the currents in the shunts are adjusted so that E_1 is numerically equal to E_2 the phase angle between them will be given by $\tan^{-1} \Delta E/E$ if the triangle is assumed to be a right-angle triangle.

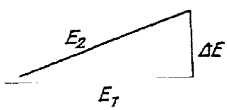


FIG. 82.—Vector triangulation.

The ratio of the two impedances will not be given, however, by the ratio of E_1 to E_2 because there is no certainty that equal currents are flowing through the shunts, and it will be necessary to connect the shunts in series and to compare the ratio of the volt drops in this series condition, in order to arrive at the ratio of the two impedances. The experiment just described is interesting as it illustrates that a phase difference often exists between the current terminals and the potential terminals of so-called non-inductive resistances, and this difference of phase can be readily determined in this manner in resistance standards of the same value. If the ohmic values of the shunts are not the same, means must be taken to adjust the currents in the two shunts in parallel until they are inversely proportional to their resistances, when the volt drops between each pair of potential points will be very nearly equal, so that the difference voltage gives the tangent of the angle as in the above example.

When this test is carried out upon low-resistance standards involving the use of large currents, considerable care must be taken to prevent stray magnetic fields affecting the potential circuit. The near proximity of a transformer or of leads carrying large currents may have an appreciable effect upon the measurement by inducing voltages into one or other of the shunts or in the potential leads connected to the shunts. The magnitude of these parasitic e.m.f.s will depend upon the design of the shunts and upon the layout of the potential leads. Most non-inductively wound resistances are not greatly affected by the presence of magnetic fields, but sometimes the potential leads are so arranged that the volt drop between the potential terminals is in phase with the current, this condition being brought about by positioning the potential leads in such a way that the field of the shunt itself brings about a compensation for self-inductance. In some shunts where this has been done, there may be the danger of an external field causing a further effect upon the potential leads which will seriously alter the compensation made with respect to the shunt alone. This is the objection to some designs of potentially compensated non-inductive shunts.

Measurement of the Ratio and Phase Angle of a Current Transformer.—A further example of the measurement of a small angle is in determining the phase relationship between the primary and secondary currents of a transformer. In the case of a current transformer this angle is very small and the method cannot be used with much success for high-grade precision transformers because the angle to be measured may be less than the phase angle of the resistance standards used in the measurement. The method is, however, applicable to any transformer in which the phase angle is more than a few minutes.

The circuit should be arranged as in Fig. 83. Non-inductive resistance standards are placed in series with the primary circuit and with the secondary circuit, so that the current flowing in each circuit can be measured by the volt drop in the respective resistance standard. It is important to use resistance values inversely proportional to the nominal currents in the two circuits. For example, if the primary current were 100 amperes a resistance of 0·005 ohm would be a convenient value

for the primary circuit. If the ratio of the transformer were 100/5 so that the secondary current would be 5 amperes, a resistance of 0·1 ohm would be required so that the volt drop on both resistances would be nearly equal. By connecting two of the potential points on the standard resistance as shown in Fig. 83, the two currents and their difference can be measured. If the volt drop on r_1 is E_1 and the volt drop on r_2 is E_2 and the difference voltage measured between the other two potential points of the resistances is ΔE, since E_1 and E_2 will be nearly equal the phase angle between them will be given by $\tan \psi = \frac{\Delta E}{E_1}$.

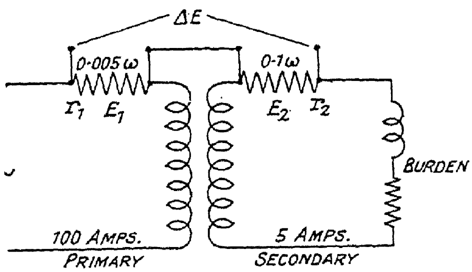


FIG. 83.—Current transformer error determination by vector triangulation.

It is essential to see that the polarity of the two windings is correct so that ΔE is the difference and not the sum of E_1 and E_2.

For this measurement it is convenient to have a special type of non-inductive resistance standard for the secondary circuit in which the position of the potential point can be varied so that the value of E_1 and E_2 can be made exactly equal under all conditions of test. This is achieved by making the value of the non-inductive shunt 1 per cent. higher than its nominal value and having two additional potential terminals brought out to a slide wire. The slide wire is arranged to shunt across a portion of the standard resistor from 1 per cent. below its nominal value to 1 per cent. above. This is shown in Fig. 84.

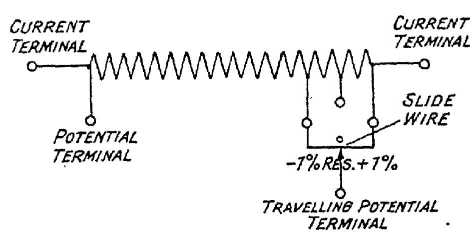


FIG. 84.—Special standard resistance for vector triangulation.

With this arrangement the travelling potential point can be moved along the slide wire until a point is found at which E_2 has the same voltage value as E_1.

The slide wire is calibrated in per cent. plus and minus the nominal value. The ratio of the primary current to the secondary current is given, therefore, directly as a percentage as indicated by the resistance setting.

Ratio and Phase Angles of Potential Transformers.—The ratio and phase angle of potential transformers can be measured in an exactly analogous manner. In this case the circuit should be arranged as in Fig. 85. Potential dividers or volt ratio boxes should be connected across the primary and secondary windings of the transformer with tapping points

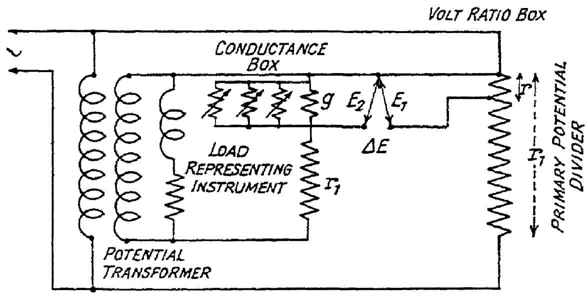


FIG. 85.—Potential transformer error determination by vector triangulation.

giving a voltage of about 1 volt on each side. That is, the resistance ratios of the volt boxes should be proportional to the transformer ratio.

The two appropriate points should be commoned between the windings so that the two voltages E_1 and E_2 and the difference voltage ΔE can be measured, thus giving the phase difference between the primary and secondary voltages. It is essential to see that the polarity of the winding is correct so that ΔE is the difference and not the sum of E_1 and E_2. If E_1 and E_2 are made equal by a variable tapping on the volt ratio box in the secondary circuit the ratio of the transformer will be given directly in terms of this setting as in the case of the current transformer just discussed. The phase difference will be given by $\tan \varphi = \frac{\Delta E}{E_1}$. The only limitation to this

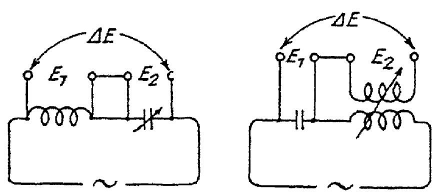
method is the difficulty of obtaining very accurate volt ratio boxes for high voltages. For voltages not exceeding 5,000 the method gives very good accuracy. The most convenient type of variable volt ratio box is one using a conductance box in series with a fixed resistance.

If r_1 is the fixed resistance and g the conductance, the ratio R is given by $R = gr_1 + 1$. Thus if the ratio of the primary volt box is R_1 the transformer ratio will be $\dfrac{R_1}{gr_1 + 1}$.

In practice g need not go down to zero but can vary over a small limited range to give a fine degree of subdivision. It should be noted that the constant term 1 is automatically eliminated from the ratio by merely altering the reading of the conductance dial by one unit. For example, in a volt ratio box for 100 volts the fixed resistance r_1 might be made 10,000 ohms. The minimum value of g would then be made 0·0098 mhos or 102·14 ohms. This would give a voltage ratio of 1 to 99. The conductance box could then add conductance in parallel up to a maximum of 0·0100 mho. This would give a ratio of 1 to 101. If the conductance box had three dials, consisting of ten steps of 0·000001, ten steps 0·00001 and one step of 0·0001 mho, the ratio could be varied 1 per cent. either side of its 1 to 100 value in steps of 0·01 per cent. The conductance setting at 1 to 100 would be 0·0099 mho. The first dial consisting of two studs would be engraved 99, 100, the second dial 0, 0·1, 0·2, etc., the third dial 0, 0·01, 0·02, etc. The ratio would then be direct reading in per cent. of the nominal value of 100/1.

The measurement of small phase angles is often required in connexion with reactive circuits. In this case, it is often the departure from true quadrature, and not the departure from zero phase angle, which is required. For example, in testing a condenser, the losses depend upon the relationship of the voltage across the condenser and the current through the condenser not being exactly 90° apart. It is much more difficult to measure a phase angle of 89° than of 1°, so that with such a measurement it is better to measure the departure from quadrature rather than the actual phase angle between the voltage and current. There are two main standards of quadra-

ture, viz. the mutual inductance and the air condenser. A first-grade mica condenser at power frequencies is also a high-grade standard of quadrature. In both the condenser and the mutual inductance, the e.m.f. induced in the secondary of the latter or the volt drop across the former is very closely in quadrature with the current passing through the circuit. When, therefore, it is necessary to measure a phase angle which is nearly 90°, it is most conveniently done by connecting a condenser or a mutual inductance in series, as shown in Fig. 86(*a*) and Fig. 86(*b*), in such a way that the difference between the volt drop on the condenser and the unknown voltage, or alternatively, the difference between volt drops on the secondary of the mutual inductance and the unknown voltage, can be compared. Exactly the same rule should be followed here as in the previous example, and the two volt drops being compared



(*a*) (*b*)

FIG. 86.—Measurement of impedances having large phase angles.

should be adjusted to equality as nearly as possible. This can be done either by varying the mutual inductance or by varying the capacitance used as a standard of quadrature. As an example, the measurement of the phase angle of a loading coil may be considered. The loading coil usually has a very large phase angle so that the volt drop across the coil is nearly 90° in advance of the current through the coil. If the loading coil is placed in series with a condenser having a value such that the impedance of the condenser is very nearly equal to the impedance of the loading coil (see Fig. 86(*a*)), the voltages across the condenser and across the loading coil will be equal, but whereas the voltage across the loading coil will lead the current, the voltage across the condenser will lag behind the current, so that the difference voltage between them will be a small quantity and the vector voltages will form a very narrow

triangle. If a high-grade mica condenser, the losses of which are very small (and preferably known) is used, the phase angle of the inductance will be readily determined by measuring the three voltages, E_1 the volt drop across the inductance or loading coil, E_2 the volt drop across the condenser and ΔE the volt drop across the beginning of the inductance and the end of the condenser, *i.e.*, across the inductance and condenser together. As before, it will follow that the tangent of the phase angle of the inductance will be equal to $\frac{\Delta E}{E_1}$, or more exactly the phase angle $= 2 \tan^{-1} \frac{\Delta E}{2E_1}$, so long as E_1 and E_2 are equal. In the case of the measurement of the phase angle of a paper condenser it will probably be simplest to use a mutual inductance, the primary winding being in series with the condenser and the value of the mutual inductance so adjusted that the secondary voltage is very closely equal to the volt drop across the condenser. If one end of the secondary winding of the mutual inductance is connected to one end of the condenser, the difference voltage between the other end of the condenser and the other end of the inductance will be very small, and this will give the difference in phase angle between the condenser and the mutual inductance, as in the previous example. It must be remembered that not every mutual inductance forms a pure standard of quadrature and that all mutual inductances suffer the defect of impurity in their quadrature, the magnitude of which depends upon the method of construction used. In well-constructed mutual inductance standards the error is very small and amounts to an equivalent resistance in the primary circuit of a small fraction of an ohm depending upon the frequency. In a mutual inductance of 10 millihenries, the impurity may be equivalent to the mutual inductance having a phase defect of 10^{-4} radians at 50 cycles and this phase effect must be subtracted from the result in calculating the difference between the condenser. Care must be taken to avoid using a mutual inductance standard too near any metal in which eddy currents may be induced and so cause phase errors in the secondary voltage.

It is a great advantage when measuring small phase angles

to be able to make the measurements in rectangular co-ordinates in preference to polar form, as the calculations are simpler, and generally speaking, the precision which can be obtained is higher. In order to facilitate this class of measurement, the Drysdale potentiometer has now been fitted with a quadrature dial which allows small voltages to be added in quadrature with the main voltage given by the dials of the potentiometer. This quadrature dial has a reading of plus and minus 0·01 volt and is connected to the compensator or phase-2 winding of the phase-shifting transformer so that the current through the quadrature slide wire will always remain in quadrature with the current through the main potentiometer dial. When using the potentiometer, voltages are balanced in the normal way by adjusting the phase-shifter and the dials of the main potentiometer, the quadrature dial being set at zero. So long as the voltages under test have been arranged to be equal in the way previously described, small phase difference can be measured simply by adjusting the quadrature dial, thus avoiding the coarseness of adjustment of the phase-shifter. The tangent of the angle so measured is then given directly by the ratio of the quadrature reading to the main reading. A further feature introduced into the Drysdale potentiometer consists of a mutual inductance connected into the quadrature circuits, and this enables the phase-splitting to be carried out by means of the vibration galvanometer instead of by observing the constancy of the current through the main dials. Fig. 87 shows the schematic arrangement of the circuit. When the current in the quadrature circuit is correctly adjusted as to phase, the voltages induced in the secondary of the mutual inductance will be in phase with the voltage on the main dials of the potentiometer, and will be proportional to the frequency. It is only necessary to set the dials of the potentiometer to the appropriate values and to connect them to the secondary of the mutual inductance and adjust the phase-splitting device consisting of the resistance box in series with the condenser until these two voltages balance. The phase-shifter is then correctly excited. This balance should remain true at all positions of the transformer, since when an exact circular rotating field is produced in the stator of the phase-shifting transformer, the

induced voltage in both windings of the rotor should be equal and at right-angles at all positions. This new addition to the Drysdale potentiometer enables an accurate test to be made of the accuracy of the phase-shifting transformer at all angular positions. Experience has shown that whenever a phase-shifting transformer is introduced into the potentiometer supply the steadiness of the secondary current, and therefore of the

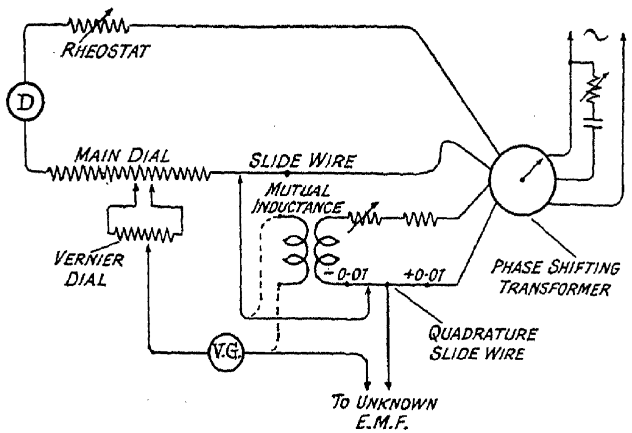


FIG. 87.—Drysdale potentiometer with quadrature slide wire.

potentiometer voltages, is impaired, so that although the phase-shifting transformer is a very convenient device for its purpose it does definitely place a limitation upon the steadiness with which an unknown potential can be balanced. It is practically impossible to build a rotating mechanism sufficiently rigid to prevent the mechanical vibrations modulating the output from the rotor. The variations are, however, less than is likely to be of importance in most industrial measurements. The most accurate arrangement of the Drysdale potentiometer is to use a reflecting dynamometer of the type shown in Fig. 51, and to check the phase-splitting by means of the mutual inductance, as described above. By this means alternating current calibrations of voltage and current can be made with errors of only a few parts in ten thousand.

NOTES AND BIBLIOGRAPHY.

[1] *Book of American Society for Testing Material Standards*, 1933, i, A34–33, 396–409.

[2] Gall, D. C., and Sims, L. G. A.: "On the Theory and Measurement of the Magnetic Properties of Iron," *J. Instn. Elect. Engrs*, 1934, lxxiv, 453–68.

[3] Press, A.: "Treatment of Harmonics in Alternating Current Theory by means of a Harmonic Algebra," *Univ. of California Publications in Engineering*, Sept. 30, 1919, Vol. 2, No. 5, pp. 139–204.

[4] Gall, Douglas C.: "A Device for separating the Harmonics of Complex Waves with special application to the A.C. Potentiometer," *J. Sci. Instrum.*, 1932, ix, 262–4.

[5] Joubert, Jules: "Sur les courants alternatifs et la force électromotrice de l'ark électrique," *J. Phys. Theor. Appl.*, 1880, ix, 297–303.

[6] Gates, B. G.: "A New Harmonic Analyser," *J. Sci. Instrum.*, 1932, ix, 380–6.

[7] Sims, L. G. A.: "Analysis of Wave Forms," *Wireless Engr.*, 1934, xl, 419–23.

[8] Sims, L. G. A.: "Incremental Permeability and Inductance," *Wireless Engr.*, 1935, xii, 8–16, 65–9.

[9] Agnew, P. G.: "A Study of Current Transformers with particular reference to Iron Loss," *Bull. U.S. Bur. Stand.*, 1911, vii, 423–74.

[10] Gall, Douglas C.: "Testing Nickel Alloys by means of the A.C. Potentiometer," *J. Sci. Instrum.*, 1932, ix, 219–22.

[11] Measurements on current transformer made by D. Connelly, at the Royal Technical College, Glasgow.

CHAPTER XI.

OTHER USES OF THE A.C. POTENTIOMETERS.

There are many uses of the a.c. potentiometer other than those which have been described in the previous chapters. These may be divided roughly into three groups. Those in which the potentiometer is used directly as an instrument for the determination of voltage. Those which combine the potentiometer principle with the bridge principle for the accurate determination of impedances, and those which use the properties of the circuits used in potentiometer work in a manner different from the two preceding methods of use.

Many investigational measurements may be included in the first group; measurements of voltage, that is, from which other electrical quantities can be determined to give the characteristics or physical condition of various circuits, apparatus or material. The exploration of stray magnetic fields and the investigation of potential distribution in complex networks where isolated voltages can be determined in their true phase relationship, are examples. Where it is not desirable to disturb the conditions of the circuit under investigation by the introduction of measuring instruments, the a.c. potentiometer is particularly useful. Such cases occur in valve amplifiers and in investigations into the performance of many types of indicating instruments. T. Spooner [1-2] has described a number of practical investigations including the measurement of amplification factors of audio frequency amplifiers and in the testing of steel for correct heat treatment by measuring its magnetic properties. The a.c. potentiometer has been used with conspicuous success for the determination of the characteristics of the Inawashire 154 kV, 220 kilometre high-voltage transmission line.[3] It has also been used to measure the mutual impedance between power and communication circuits.

In the second and third groups may be included various measurements on valve circuits, high-precision current transformer testing and a method of medical diagnosis depending on the measurement of body potentials. Various examples of these other uses to which the potentiometer can be adapted will now be considered.

Impedance Measurements.—When the potentiometer is used to measure impedance the value of the impedance is usually determined by calculation from the current and potential measurements upon the impedance under test. In most cases this consists of making a potential comparison between a known and an unknown impedance, and the value of the unknown impedance follows from the direct ratio of the potential drops. In some cases it is important to measure the current produced in one part of a circuit when a voltage is applied to another entirely different part of the circuit, or conversely to measure the potential drop on one part of the circuit when a current is flowing in another part. These relationships between different parts of a network are known either as mutual admittance or mutual impedance, and are as much fundamental constants of the circuits as the self-inductance or self-impedance of any branch of the network.

Earth Resistivity and Impedance.—An example of the above class of measurement occurs in the measurement of earth resistivity [4, 8] and impedance somewhat similar in character to those dealt with in Chapter IV. In prospecting for minerals, and in locating water and geological formations much information is to be obtained from measurements of the electrical properties of the earth. These measurements have to be made over large tracts of land, and between electrodes widely spaced. It is not possible to obtain the measurements by inserting two electrodes into the ground and measuring the impedance between them, because the contact resistance between the electrodes and the ground may be hundreds of ohms, whereas the resistance of the earth between them only a fraction of an ohm. In order to eliminate the contact resistance of these electrodes, four electrodes are used as shown in Fig. 88. Current is passed into the earth at two of the electrodes and the volt drop measured at the other two. This is similar to the measurement

of earth resistivity given in Chapter IV. If the current entering the first two electrodes is measured as well as the volt drop which the current produces at the second two electrodes, the mutual impedance between the two current and the two potential electrodes will be given by the current entering the two electrodes divided into the voltage drop across the other two electrodes.

That this mutual impedance is something different from the self-impedance between the electrodes can be easily demonstrated by moving the current electrodes nearer to the potential electrodes. The value of impedance measured in this way will in general increase asymptotically, so that the self-impedance is the same as the mutual impedance when current and potential points coincide.

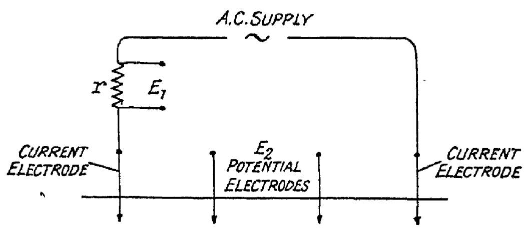


FIG. 88.—Measurement of earth impedance.

Owing to the very high contact resistance of the earth electrodes a potentiometer method of measurement is necessary to eliminate the effects of this resistance from the measurement, so that no current is taken from the potential electrodes.

If a resistance is inserted in series with the current electrodes and the volt drop upon this resistance measured as well as the volt drop between the potential electrodes, the mutual impedance will be given directly as the ratio of these two measured voltages. That is the mutual impedance $Z_M = \frac{E_2}{E_1}r$ where E_2 is the volt drop across the resistor r and E_1 is the volt drop across the potential electrodes.

If the potentiometer circuit is closely coupled to the main current circuit it will have the advantage that fluctuations of current in the circuit due to variation in the contact resistance

of the current electrode will be automatically compensated in the potentiometer circuit. Fig. 89 shows the circuit arranged to achieve this. The potentiometer is supplied from the secondary winding of a current transformer so that the current in the potentiometer circuit is always in the same ratio to the primary current, which is the current entering the ground. Since the potentiometer voltage is proportional to the current in its circuit, this voltage will be directly proportional to the current entering the ground, and will therefore follow any variation in this current due to fluctuation of the supply or the resistance of the electrode contacts.

The type of potentiometer shown is a simple Larsen com-

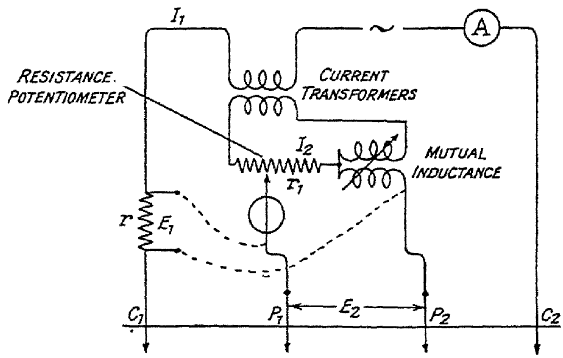


FIG. 89.—Measurement of earth impedance.

bination as this lends itself very well to this purpose. The current in the resistance potentiometer and the primary of the mutual inductance will be sensibly in phase with the current entering the ground. In measuring the volt drop upon the resistance r it will be balanced almost entirely upon the resistance potentiometer. The mutual inductance will only be required to compensate for any phase defect in the current transformer, the resistor r, and the resistance of the potentiometer. In measuring the volt drop between the potential electrodes in the ground, there will generally be a phase difference which will require the mutual inductance to compensate for it.

If the current in the main supply circuit is $\mathbf{I}_1$ the current

in the potentiometer circuit being I_2, $I_2 = RI_1$, where R is the ratio of the current transformer. The volt drop on the potentiometer will be $I_2(\pm r_1 \pm j\omega M)$.

If the volt drop between the two potential electrodes P_1 and P_2 is balanced when the potentiometer setting is r_1 on the resistance section and M on the mutual inductance, the mutual impedance will be $Z_M = R(r_1 \pm j\omega M)$ if there are no errors or phase defects in the current transformer.

This can be checked directly against the resistance r in the main circuit where the volt drop should be balanced by the setting $r = Rr_1$.

It will be clear from the above that mutual impedance of earth electrodes or other circuits can be measured directly by a bridge potentiometer of this type, in which current variations do not affect the balance.

Comparison of Current Transformers.—A further application of this type of measurement is for the comparison of current transformers. It is often necessary to compare current transformers for ratio and phase angle to a very high

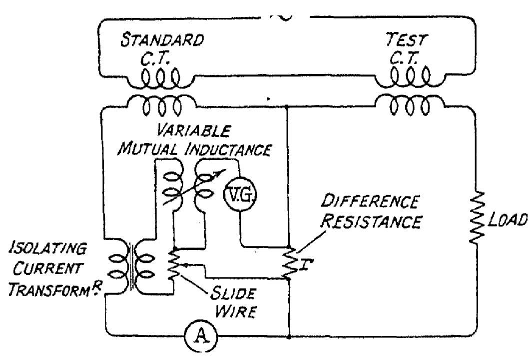


FIG. 90.—*Circuit diagram of N.P.L. method for determination of current transformer errors.*

degree of precision. One of the circuits used for this purpose is shown in Fig. 90, and Figs. 91, 92, 93 show the actual equipment as designed by the N.P.L. for this measurement.[5]

The primary of the current transformer under test is connected in series with the primary of the standard current

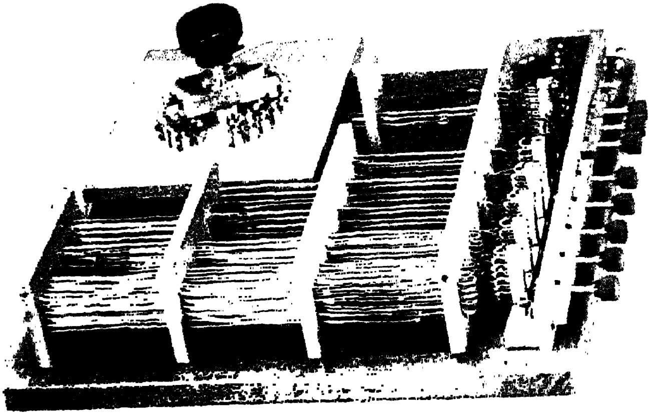


FIG. 91.—Artificial load unit for circuit of Fig. 90.

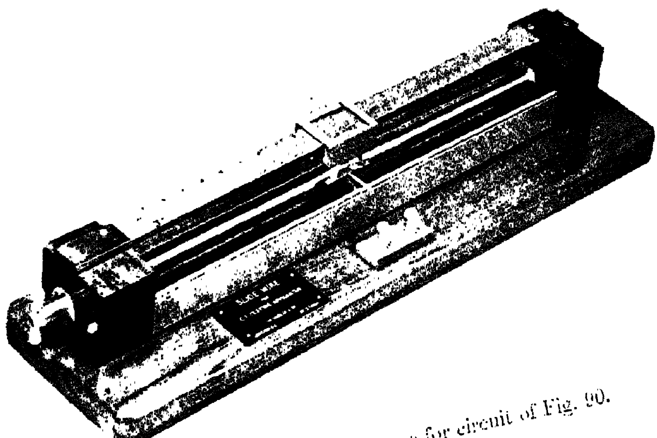


FIG. 92.—Slide-wire resistance for circuit of Fig. 90.

To face p. 166]

transformer, the ratio and phase angle of which are known. The two secondaries, usually of 5-ampere range, are also connected in series so that the current will flow round them in the same direction. In the secondary circuit is connected the load into which the transformer must work and a second current transformer of 5/5-ampere range. This transformer serves to isolate the potentiometer which is used to measure the "difference" current of the two secondaries. If the transformer under test had exactly the same characteristics as the standard transformer, there would be no difference current in the cross-circuit resistance r shown in Fig. 90 because for the same primary currents the same secondary currents would flow, but if there is a difference between the two transformers, the secondary currents will be different, and the difference current will flow through the cross-circuit. By measuring the difference current by the volt drop upon a known resistance, the difference of both ratio and phase angles between the transformer can be found.

The volt drop upon the resistance portion of the potentiometer will be in phase with the current in the secondary of the standard transformer and the voltage in the mutual inductance will be in quadrature. When balanced, the reading of the resistance gives, therefore, the difference in magnitude of the two secondary currents and the mutual inductance gives the phase difference.

If the isolating current transformer has an even ratio, the current in the potentiometer circuit will be the same as in the secondary circuit and the ratio error will be given by $100\frac{r_p}{r_1}$ per cent. when r_p is the resistance of the potentiometer setting and r_1 the "difference" resistance. The phase difference will be given by $\tan^{-1}\frac{\omega M}{r_1}$ where M is the balance setting of the mutual inductance in henries.

A further example of this type of potentiometer measurement is the calibration of a potential transformer.

This circuit is shown in Fig. 94.

The primary of the potential transformer is connected in parallel with a high resistance in series with which is the poten-

tiometer. The current through the potentiometer is almost exactly in phase with the primary voltage because the self-inductance of the mutual inductance is negligible with respect to the high resistance in series with it. The potentiometer is used to balance the reversed secondary voltage of the potential transformer which is carrying its normal load. If the secondary voltage were exactly in phase with the primary voltage, the setting of the mutual inductance would be very small or zero, and the ratio would be given by the ratio of the potentiometer resistance to the total resistance, that is, the ratio of the transformer would be given by $\frac{r_p}{r_1} = R$.

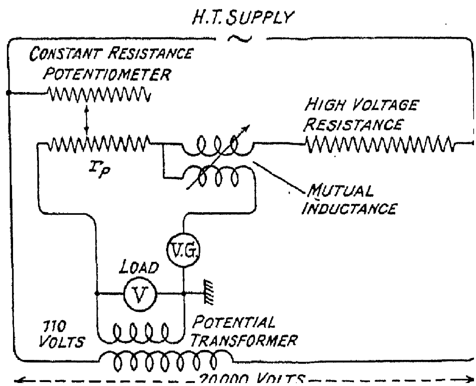


FIG. 94.—Determination of errors of potential transformer.

The value of r_1 is kept constant, using a resistance potentiometer of the constant resistance or substitution type. If the secondary current is not in phase with the primary current, it will be necessary to adjust the mutual inductance to balance the secondary voltage. In this case the phase difference will be given by $\tan^{-1}\frac{\omega M}{r_p}$.

The Turns Ratio of Power Transformers.—In the manufacture of power transformers, it is usual to make an open circuit test of the transformer at comparatively low voltage for the purpose of checking the turns ratio. This can be done by exciting the primary or secondary winding and making use of a potentiometer to compare the volt drops upon the two wind-

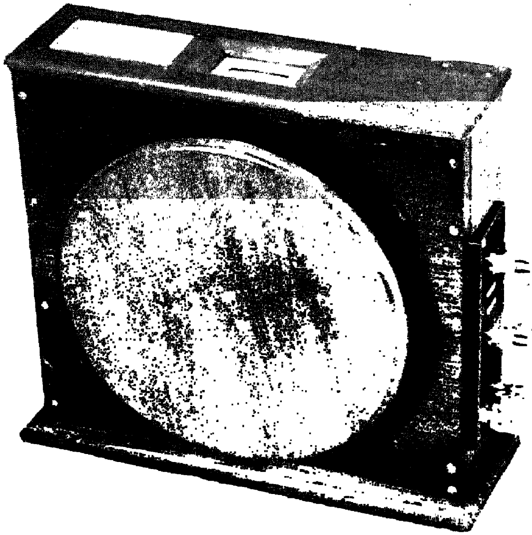


FIG. 93.—Variable mutual inductance for circuit of Fig. 90.

FIG. 96.—Apparatus for determination of turns ratio of power transformer.

To face p. 168]

ings. Fig. 95 shows the theoretical circuit of such an instrument. For workshop purposes, it is essential that it should be compact, portable, and require no calibrating. At the same time a high degree of subdivision is necessary to cover individual requirements. Fig. 96 shows an actual instrument arranged for this purpose.

The point of interest in the instrument is the use of a conductance box for obtaining the potential variation. The circuit is very similar to the potential transformer test described in Chapter X. A high resistance is shunted across the supply, which also feeds the primary of the transformer.

In series with the high resistance r is a conductance box so that the ratio of the potentials at the junction will be given

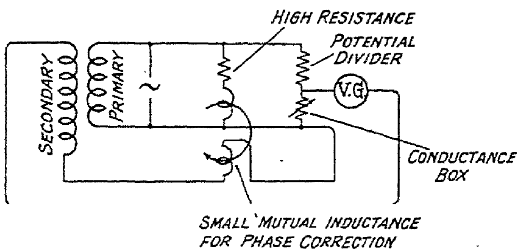


FIG. 95.—Determination of turns ratio of power transformer.

by $\dfrac{r_1 + 1/g}{1/g}$ where g is the conductance in mhos of the conductance box. That is, the ratio R is given by $r_1g + 1$ which will be direct reading in terms of g, if the conductance box is made to read 1 when set at 0. That is, the engraving of the units dial is displaced one stud.

A mutual inductance with its primary in series with a separate high resistance is also connected across the transformer winding. This is to compensate for the phase difference between the primary and secondary voltages. The secondary winding of the mutual inductance is in series with the galvanometer which is arranged to balance the volt drop upon the conductance box against the transformer secondary voltage. A wide range of transformer turns ratios can be covered with a high degree of precision. The exciting voltage used is usually much below the normal working voltage of the transformer

so that the primary impedance drop due to the exciting current is negligible.

Measurement of Leakage Impedance.—The leakage impedance of the transformer can be determined if the true turns ratio is known, since the difference between the primary voltage and the open-circuited secondary voltage corrected for the turns ratio will be the volt drop due to the leakage impedance of the primary. The indicated ratio and phase defect therefore gives the difference from the true turns ratio.

The Measurement of Magnetic Fields.—In measuring the strength of alternating magnetic fields by means of the a.c. potentiometer it is usual to employ a search coil consisting of an appropriate number of turns occupying a suitable area. In high intensity magnetic fields, such as occur in iron circuits, the search coil may be of a thin flat shape wound on glass or other rigid insulator, and having a cross-sectional area of only a few square millimetres, whereas when measuring field strengths in air such as in some methods of geophysical prospecting, the search coil may be wound on a large wooden framework and have thousands of turns. The design of such coils depends upon the field strength to be measured and the required value of induced voltage to suit the range of the potentiometer.

The voltage induced in a coil will be in r.m.s. volts.

$$E = \sqrt{2}\pi f N \Phi' 10^{8} \text{ volts}$$

where Φ' is the total maximum value of the flux in maxwells embraced by the coil. If B' is the maximum value of the field strength in gauss and A the area in square centimetres the voltage induced by any field strength H' will be

$$E = \sqrt{2}\pi f NAB' 10^{-8} \text{ volts}$$

when N is the number of turns.

Thus in measuring a field strength of $B' = 1$ at 50 cycles in order to induce 1 volt in the search coil the area turns or N.A of the latter would have to be equal to 450,000. This means a very large coil or a large number of turns, for example a coil of 1 square metre in area requires 4·5 turns, or a coil having an area of 100 sq. cm. requires 4,500 turns. The measurement of very weak fields of less than 10^{-4} gauss involves difficulties at low frequencies, because the induced voltages are

low and at the same time the impedance of the search coil will tend to become high. The design of the most efficient search coil to give the highest sensitivity is therefore a matter of compromise. If too large a wire is used to keep the resistance low the weight and cost may be too high. The inductance of a given size of coil, and therefore the impedance, increases as the square of the number of turns, but the induced voltage only directly as the turns. Thus the sensitivity which will be proportional to the induced voltage will probably be inversely proportional to the turns, but this depends upon the impedance of the external circuit consisting of the detector and the potentiometer. In general this part of the circuit cannot be designed to suit the search coil except to the extent of matching the impedance of the detector by a transformer; hence the solution usually lies in the use of the largest possible search coil which can be constructed rigidly enough and light enough having regard to convenience of use. The number of turns can then be made such that its impedance matches the detector and potentiometer circuit.

The direction in space of a magnetic field is often required as well as its magnitude. This can be determined very easily by rotating and tilting the coil until a zero voltage position is found if the field is not polarized spherically.

Polarized Magnetic Fields.—Most alternating magnetic fields which extend to any appreciable distance from their source become polarized, so that at any given point the magnetic field is no longer rising and falling sinusoidally about a zero value, but is never zero, but only passes through a minimum value. This is due to the superposition of two fields displaced in time phase. The second field may be much smaller in magnitude than the first and may arise from secondary inductive effects in neighbouring conductors, or from the superposition of fields arriving by paths of different magnetic properties.

The effect of the superposition of a number of alternating magnetic fields of different time phase and different direction is to produce a resultant field which rotates in such a way that its magnitude traces out a lemniscate in time, but whose magnitude measured at different orientations in space traces an ellipse, or ellipsoid if in more than two dimensions. A search

coil in such a magnetic field will have a voltage induced in it dependent upon the flux normal to the plane of the coil. If the curvature of the field is negligible, the search coil can be used to measure the field strength. The phase and magnitude of the induced voltage will change as the coil is rotated. The phase will turn through 360° with a complete rotation of the plane of the coil. The voltage will rise and fall as the locus of an ellipse.

If two fields only of different direction and time phase are superposed, the resultant will be plane polarized and if the coil is tilted into the plane of polarization no flux will cut it.

The plane of polarization can be determined by turning and tilting the coil, but in many ways it is simpler to measure the components of the induced voltage on three coils mutually at right angles. The voltage induced in the three coils can then be measured at one setting of the coil. This gives a complete picture of the field in three dimensions.

In exploring the magnetic field distribution derived from such measurements upon a straight line traverse across the field it is convenient to plot the inphase components and the quadrature components of each plane in separate curves. The resultant values can be derived at any point if desired.

If $a_1 + jb_1$ is the induced voltage in the first coil
$a_2 + jb_2$,, ,, ,, ,, second coil
$a_3 + jb_3$,, ,, ,, ,, third coil

in an unpolarized field the ratio of a to b will be the same in each voltage, but in a polarized field this ratio will be different in each case.

The resultant of the induced voltages would be

$$\mathrm{E} = \sqrt{a_1^2 + a_2^2 + a_3^2} + j\sqrt{b_1^2 + b_2^2 + b_3^2}.$$

The numerical value of E will be

$$\mathrm{E} = \sqrt{a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2}.$$

The field strength would be calculated from

$$\mathrm{B}' = \frac{\mathrm{E}}{\sqrt{2}\pi f \mathrm{NA}} \cdot 10^8 \text{ gauss.}$$

(The time phase of B′ will be 90° in advance of the induced voltage.)

If the r.m.s. value of B is required

$$B = \frac{E}{2\pi f N.A} 10^8 \text{ gauss.}$$

The spatial direction of B will be given by the projection of the resultant on to three co-ordinate axes. The inphase component is made up of a_1, a_2 and a_3. Projection on to the axis 1 will give the angular direction of the inphase component with respect to axis 1. This will be

$$\cos \phi = \frac{a_1}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

and with respect to axis 2

$$\cos \psi = \frac{a_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

and with respect to axis 3

$$\cos \theta = \frac{a_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}}.$$

In a polarized field the space angle of the inphase components will not be the same as that of the quadrature components. It must be calculated separately, in the same way.

In the case of the measurement of strong magnetic fields, the impedance of the search coil does not usually restrict sensitivity, and very fine wire can often be used with as many turns as voltage requirements demand.

The Measurement of Electrostatic Fields.—The measurement of electrostatic fields by means of the low voltage, low impedance a.c. potentiometer is seldom possible because the size of the electrodes will have too disturbing an effect upon the field measurement.

In order to obtain sufficient sensitivity to measure the voltage gradient between two points in an electrostatic field, it would be necessary to insert at these points two electrodes, the capacitance between which would be not less than would give a circuit impedance of say 50,000 ohms. At 50 cycles this would involve a value of about 0·06 microfarad. This would necessitate two very large plates very close together. If 1 mm. apart, the two plates would have to be about 2·5 metres square. This is clearly out of the question, as they could not be easily

supported and would probably completely alter the dielectric field into which they were placed. A very high impedance device is essential for measurements of the electrostatic field. The field distribution is best plotted by finding equipotential points rather than by measurements between different potential points, the latter method causing an alteration of field distribution.

Mutual Impedance Potentiometers.—It has been pointed out that the inherent limitations to the precision obtainable by the a.c. potentiometer are in the steadiness of the supply, both as regards voltage and frequency. In many cases magnitude can be measured with a precision superior to the steadiness of the supply due to the compensation between the variations of the voltage measured and that used for measuring. In effect this means that the potentiometer is arranged to resemble a bridge in regard to its functioning. In many cases it is a definite gain to design the circuit so that this condition is rigidly fulfilled.

A bridge circuit usually measures the impedance properties of one of its arms in terms of the others by a balance method, and the condition of balance is independent of fluctuation in the supply. If these properties have to be determined under certain current-carrying conditions, then the current has to be determined by other means, such as the insertion of a current-measuring instrument into the bridge or deduced from the voltage across some part of, or the whole of, the circuit.

The potentiometer measures the potential upon any relevant part of the circuit under test and the impedance properties of the circuit are deduced therefrom.

The basic distinction between the use of a bridge and the use of a potentiometer is that in the former case the current through the circuit under test is part also of the bridge current and dependent upon the adjustment of the other bridge arms, whereas in the case of the potentiometer, the current in the circuit under test is undisturbed by the potentiometer.

By combining a bridge and a potentiometer, circuits can be measured with the advantages of both systems. That is, the measurements are made by potential measurements without taking current from the circuit tested and at the same time

the condition of balance is independent of fluctuations in the supply.

Many forms of bridges are in effect potentiometers and can often be used for this purpose, since the potentials between the corners of the bridge can be varied in both phase and magnitude by adjustment of the arms. This principle was employed by Willans [6] for producing a voltage of known magnitude for measuring the amplification of transformer coupled valve circuits. The circuit is shown in Fig. 97. The four-arm bridge shown at the left-hand side of the figure is supplied with a.c.

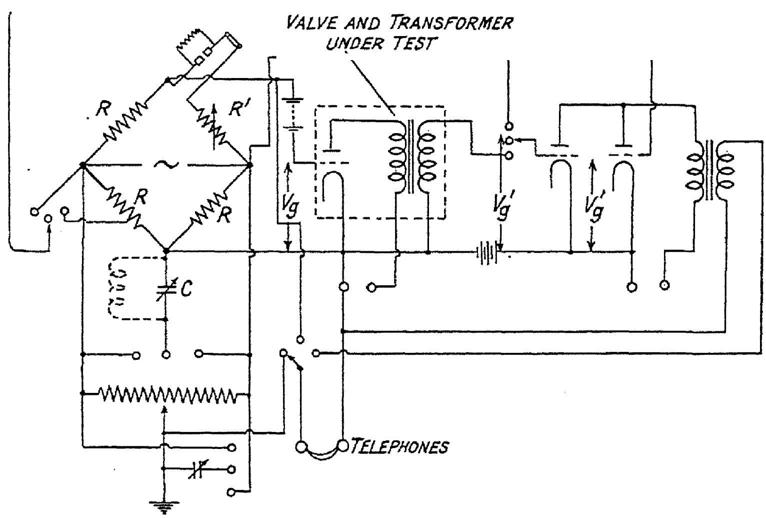


FIG. 97.—Determination of amplification of transformer coupled valve circuit.

across its vertical diagonal. The voltage across the horizontal diagonal can be controlled by adjusting the variable arm of the bridge. This voltage can then be applied to the grid of the last valve or to the input of the transformer stage. The ratio of the voltage necessary to give the same overall output allows the amplification of the transformer coupled stage to be determined. The ratio of these two voltages is easily calculated from the known values of the bridge arms.

The resistance and capacitance at the lower half of the bridge is for balancing earth capacitance. The capacitance C can be transferred from one arm to the other as necessary.

The voltage ratio is given by

$$-\frac{Vg'}{Vg} = \frac{R' + R}{R' - R - j\omega R^2C}$$

or

$$= \frac{R' + R}{R' - R + \omega^2R^3C^2 + j\omega R^2C}$$

according to whether C is across the left- or right-hand bridge arm respectively.

A Medical Diagnostic Potentiometer Bridge.—An interesting example is the application of the potentiometer bridge to the measurement of the impedance of the human body, for diagnostic purposes. The circuit of the potentio-

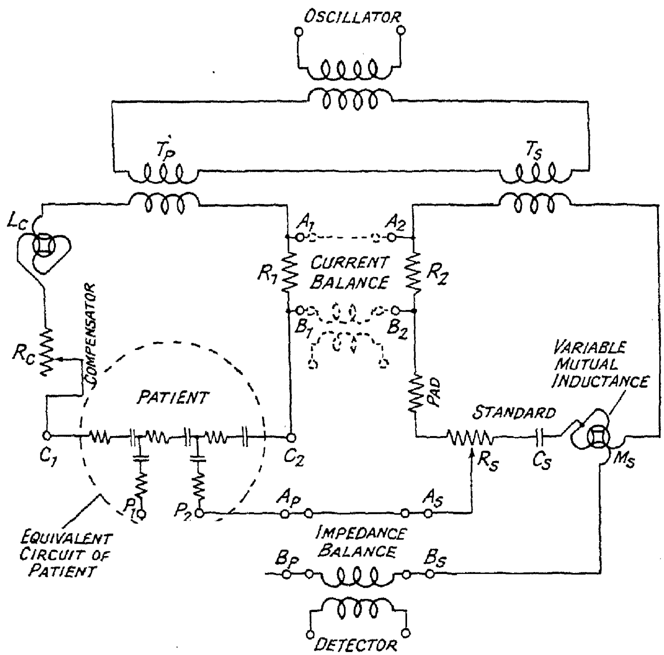


FIG. 98.—A medical diagnostic potentiometer.

meter bridge supplied by the General Radio Company is shown in Fig. 98. The body is shown as an equivalent circuit composed of impedance made up of a series of condensers and resistances. Current is fed into the body by electrodes C_1, C_2

in contact with the skin, usually the forearms. The potential difference between any points P_1, P_2 on the surface of the body is balanced by means of what is in principle a Larsen potentiometer supplied from a transformer in series with the transformer supplying the body. It is interesting to note the use of a condenser in series with the Larsen circuit. This allows the adjustment of the phase of the current, and therefore the phase of the volt drop upon the resistance potentiometer and mutual inductance to agree with the current flowing in the body, so that the resistance and reactance components of the measured impedance are direct reading. The potentiometer contact-points on the body are equivalent to a condenser and resistance in series, forming a circuit of high impedance. As the voltage which may be applied to the body must be limited to avoid shock, a sensitive detector is necessary. The frequency employed is of the order of 10,000 cycles per second. The advantage of the potentiometer bridge for the measurement is that the great variability of the surface resistance of the body electrodes does not affect the values of the impedance measured, as is the case with a simple bridge when used for the purpose.

Method of Adapting the Co-ordinate Potentiometer to the Testing of Precision Current Transformers.—The range of voltage reading required for testing precision current transformers is only about 5 millivolts on both the inphase and quadrature potentiometers. It is also desirable that the measuring voltages should be closely coupled to the circuit under test.

These requirements are easily achieved by the circuit arrangement shown in Fig. 99, using the standard co-ordinate potentiometer without the dynamometer or standard mutual inductance; but using a small fixed mutual inductance and low-resistance shunt.

The current flowing in the potentiometer circuit is only 0·0001 ampere. This arrangement requires only the addition of the 0·001 ohm shunt and 200 microhenries mutual inductance, and the isolating current transformer to the standard instrument. Owing to the very small current taken by the quadrature potentiometer, the phase of the quadrature current is almost exactly in quadrature, the phase defect being governed

by the self-inductance of the secondary circuit in relation to its resistance. This is about 200 microhenries and 40 ohms, so that the phase defect from true quadrature is $\frac{\omega L}{r} = \tan^{-1} 0{\cdot}0157$ or about 5 minutes. This is negligible in its effect upon the

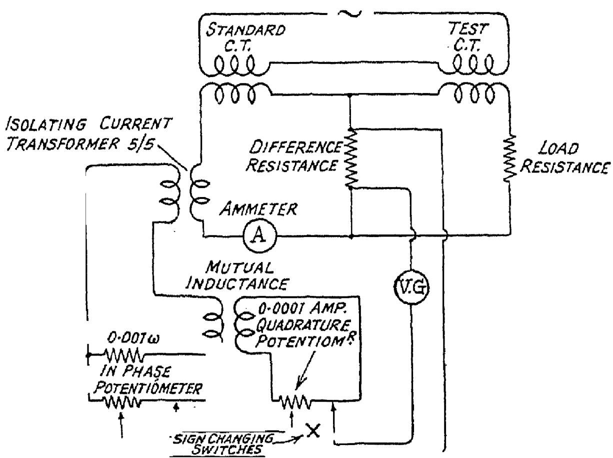


FIG. 99.—Circuit for testing precision current transformers.

determination of the already small quantity which represents the error of ratio and phase, but can be compensated if desired so that the currents in the two potentiometers are in exact quadrature, by inserting a small inductance in series with the inphase potentiometer.

The Method of Injection.—In all the measurements of potentials so far described, the principle employed has been to balance an unknown voltage against a known voltage. This method makes it essential to have a voltage of which the phase and magnitude are known and can be controlled. In some classes of measurement this is not easy to achieve and the method of potential injection may be employed. If a voltage is acting in a circuit and another voltage is injected into that circuit, the total voltage will be modified and will be increased or reduced according to the phase of the injected voltage. If now the magnitude of the injected voltage is adjusted until the combined voltage is a minimum but not necessarily zero,

this minimum value must represent the component of the first voltage which is in quadrature with the injected voltage. If the injected voltage is changed to one at right angles or in quadrature with its first position and again adjusted until the combined voltage is again a minimum, this minimum value must now be the component of the first voltage which is in quadrature with the injected voltage. This will be evident from Fig. 100(a). If OA is a voltage vector acting in a circuit into which a voltage having a phase relationship of φ is injected, the addition of the two voltages will produce a resultant voltage whose magnitude will clearly be a minimum when the locus of the two voltages passes nearest to the origin. This will occur when the resultant is at right angles to the phase of the injected voltage. The minimum resultant is therefore the quad-

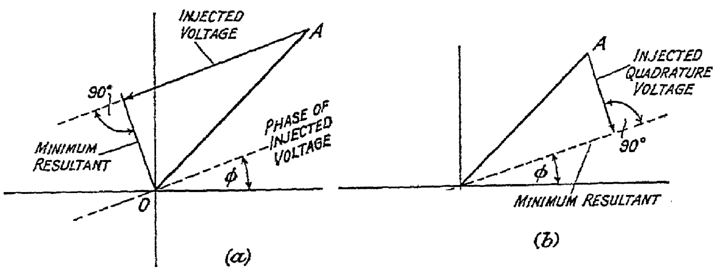


FIG. 100.—Measurement by voltage injection.

rature component of the first voltage to a new co-ordinate system inclined at the angle of the injected voltage.

By repeating the process with an injected voltage at right angles to the first injected voltage, the inphase component with respect to the new co-ordinate system can be found. This is indicated in Fig. 100(b). In this way, the components of an unknown voltage can be measured by injecting into the circuit two unknown voltages at right angles with each other and measuring the resultants. This principle has been employed in connexion with the measurement of surface potentials in geophysical prospecting. Fig. 101 shows a valve voltmeter equipped with the injector circuit for analysing the phase and magnitude of the voltage measured upon the valve voltmeter. The injecting voltage from the same source of supply as that

which is being measured upon the voltmeter can be picked up by means of an aerial or from the earth surface potentials. The injecting circuit consists of what is, in principle, a small Larsen potentiometer, from which components of voltage at right angles to each other can be injected into the voltmeter circuit. It will be evident that this method of potential injection, as a means of analysing the phase and magnitude of an unknown voltage, can only be employed when the impedance of the injector circuit is negligibly small in relation to the circuit into which the voltage is injected, otherwise the injector circuit will affect the original voltage. The advantage of the method is

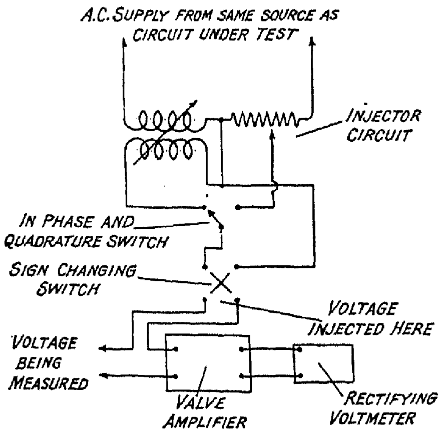


FIG. 101.—Measurement of surface potentials in geophysical prospecting.

that it is unnecessary to know the magnitude of the injected voltages but only the phase angle between them. There are many sources from which voltages of known phase but unknown value can be derived, especially where measurements at audio frequencies are required.[7]

The Geophysical Ratiometer.—The geophysical ratiometer [8] is a special form of instrument used for determining by a potential balance the distribution of surface potentials upon the earth in prospecting for minerals and in certain geological investigations. It is, therefore, a specialized form of a.c. potentiometer.

Current at about 500 cycles is fed into the ground by means

of electrodes at a considerable distance apart. The volt drop between these electrodes distributes itself according to the conductivity of the earth. Thus a series of equipotential lines are formed on the earth's surface normal to the current path between the electrode.

If the earth conductivity is not uniform the volt drop will not be uniform and the spacing of the equipotential lines will not be equal. The ratiometer permits the measurement of the spacing of the equipotential lines. It consists of two high resistance arms, one of which is variable. Three steel electrodes are pushed into the ground at uniform intervals in a straight line, the ratiometer resistances are connected between the two outer electrodes, and the centre electrode taken to the junction of the two resistances through a sensitive detector amplifier. The circuit is shown in Fig. 102. If the volt drop on the earth's surface is uniform, then there will be no current flowing in the centre tapping circuit when the two arms of the ratiometer are equal. If the volt drop is not uniform, then the variable arm of the ratiometer must be adjusted until no current flows in the centre tapping circuit. The ratio of the two resistances gives the potential ratio upon the earth's surface.

In practice, however, it is not possible to obtain a balance in the detector by simple resistance ratios. The reason for this is that the surface potentials upon the earth are invariably polarized to some degree. In highly conductive areas near mineral deposits, the surface potentials are often almost circularly polarized. This means that there is a very big difference in phase between the two surface volt drops which are being compared. To allow the phase difference to be corrected the condenser shown in Fig. 102 is introduced. This is variable and must be adjusted until balance is obtained. The ratiometer then shows the inphase potential ratio and the quadrature potential ratio between the two outer and the centre electrodes. A clear picture of the nature of the surface potential can be obtained if the voltage distribution is represented by two systems of equipotential lines [9] at equal volt drops apart. First there is a system of lines of equal inphase potential. Along any one of these lines there will be no difference of inphase voltage, but there may be large difference of quadrature

oltage. Thus, there is a second system of lines of equal uadrature voltage which will cross the inphase lines. The ifference of potential between any two points on the earth's ırface will be given by the number of inphase equipotential nes between those points plus the number of quadrature equiotential lines between them. This will be a vector value of hich the two line systems give the components. If the uadrature lines are very faint, that is the quadrature volt rop per foot of surface is very small, then the inphase equiotential or E.P. lines are very easily plotted by finding suc-

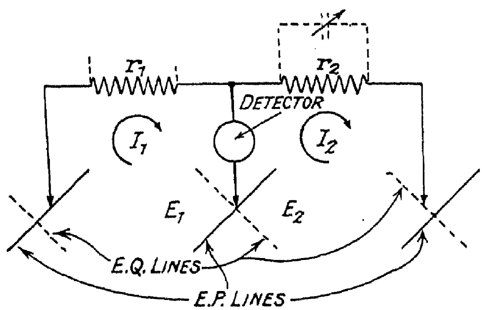


Fig. 102.—The geophysical ratiometer. (A. Broughton Edge.)

ɔessive points between which there is no voltage difference. Where the quadrature equipotential, or E.Q. lines as they are ɔalled, are strong there are no successive points at which there s no voltage difference. These conditions are frequently ıssociated with highly mineralized areas and are therefore of considerable importance. In these areas the E.P. and E.Q. lines can be plotted by means of the ratiometer.

The theory of the balance is as follows. As indicated in Fig. 102 the two voltages acting in the circuit are E_1 and E_2. These are the surface potential differences between the main and the two outer electrodes. These are in the same direction and send circulating current I_1 and I_2 round the ratiometer.

The detector will be balanced when $I_1 = I_2$.

$$I_1 = \frac{E_1}{Z_1} \text{ and } I_2 = \frac{E_2}{Z_2}$$

$$\therefore \frac{E_1}{E_2} = \frac{Z_1}{Z_2}.$$

To determine the relative phase of E_1 with respect to E_2, let $E_1 = a_1 + jb_1$ and $E_2 = a_2$ since this is the reference vector. If Z_2 is a simple resistance, r_2,

$$\frac{a_1 + jb_1}{a_2} = \frac{r_1 + jx_1}{r_2}.$$

$$\text{Voltage Ratio Inphase} = \frac{a_1}{a_2} = \frac{r_1}{r_2}.$$

$$\text{Quadrature} \quad = \frac{b_1}{a_2} = \frac{x_1}{r_2}$$

$$= \frac{\omega C}{r_2(1/r_1{}^2 + \omega^2 C)}.$$

When the capacitance is transferred to the other arm in order to obtain a balance the ratios are as follows:

$$\text{Voltage Ratio Inphase} \quad = \frac{a_1}{a_2} = \frac{r_1}{r_2}$$

$$\text{Voltage Ratio Quadrature} = \frac{b_1}{a_2} = r_1 \omega C.$$

This latter ratio gives the spacing of the E.Q. lines in terms of the E.P. lines.

In order to make a direct comparison between the E.P. and E.Q. lines, one arm of the ratiometer is made capacitive only. The detector can then only be balanced when one electrode is on one E.P. line and the other on an E.Q. line. In this way the angle of intersection of the two-line system is determined.

To follow an E.P. line over a highly polarized area, two electrodes are inserted at suitably spaced points on the line. The third electrode is then moved about and the ratiometer adjusted simultaneously until a point is found where balance is obtained with no capacitance in the ratiometer. This means that the third point so found has no phase difference from the first two points, and is therefore on the same E.P. or for that matter same E.Q. line as the first two electrodes. A very exact delineation of the line systems can be made in this way. The operator uses the third electrode and also operates the ratiometer in actual field work. An electrode spacing of about 50 to 100 feet is convenient, and lines can be plotted at the rate of about half a mile an hour under highly polarized conditions.

Radio Frequency Potentiometers.—Radio frequency potentiometers have been devised, but [10–11] the scope of measurement in this field is limited by a number of practical considerations. The chief of these is the difficulty of attaching the potentiometer to the circuit without altering the conditions of the circuit. Joining the potentiometer leads to a high-frequency circuit will alter the capacitance to earth of the point of attachment. The capacitance between the leads may also have a disturbing effect upon the circuit, and the length of the leads may be sufficient to act as aerials, either radiating or receiving energy, and thereby disturbing actual working conditions. There is also the difficulty of introducing a detector into the leads of such a nature that it will detect potential differences between the circuit under test and the potentiometer. Any sensitive detector must include some form of amplifier, and this will have a large self- and earth capacitance. With the exception of long waves, any potentiometer in the same sense as heretofore described is not a suitable instrument. For short-wave work any scheme of balancing potentials as a means of measurement is at present technically unworkable.

The attenuator is, however, widely used as a form of radio frequency potentiometer. It acts as a device producing a known potential, *not* for balancing an unknown potential.

The attenuator consists of an impedance network which attenuates a known input voltage in a definite ratio. These ratios are usually expressed in logarithmic form or in decibels. The most convenient form is a "ladder" of resistances so chosen that the potential is attenuated in definite steps along the ladder. Fig. 103 shows the circuit arrangement of an

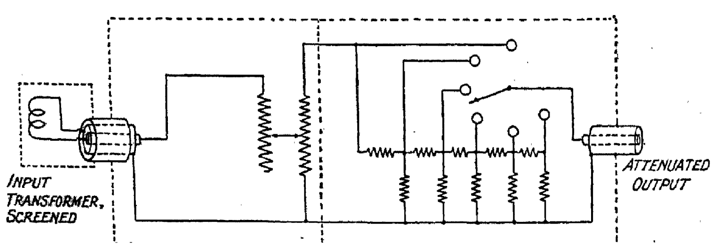


FIG. 103.—Screened attenuator.

attenuator giving a voltage attenuation between the input and output of successive steps of 0, 10, 20, 30, 40 and 50 decibels, with a continuously variable section of 10 decibels.

Considerable care is necessary in the construction of attenuators to screen the output from direct pick-up from the input, especially when the ratio of the attenuator is large. It is not possible to make a large potential ratio on two simple resistors at high frequency for this reason, and large attenuation can only be obtained by a series of meshes each attenuating into the next.

There must be no appreciable mutual inductance or mutual capacitance between successive stages, if an accurate ratio of attenuation is to be obtained. It will be noticed that these requirements are entirely different from those pertaining to a low-frequency potentiometer, where high precision over a continuous voltage gradient is required. In the high-frequency attenuator, a very large voltage ratio of known value is the chief requirement. The attenuator is used for testing amplifiers by injecting a small known voltage and observing the output of the amplifier. The input to the attenuator is usually measured by some suitable instrument such as a thermal milliammeter giving the product of the current and its known resistance (thus giving a known input voltage). Alternatively a valve voltmeter may be used across the input to measure the voltage. The output voltage is then known in terms of the input voltage by the ratio of attenuation brought about by the attenuating network. One side of the radio frequency attenuator is generally earthed, but the point of earthing is of great importance, otherwise the screen may pick up potentials which will appear at the output terminals. The screen should be earthed at one point only, at the output terminals, which are usually in concentric plug form, the outer screen being earthed.

It is possible to arrange a co-ordinate system using two attenuators with phased inputs in order to control the phase of the output voltage, for the purpose of injecting high-frequency voltages of controllable magnitude and phase into a circuit.

High-frequency attenuators are also constructed of series condensers.[12] A small condenser in series with a large one can

be used to attenuate the voltage in a known ratio so long as the condensers are very carefully screened and preferably non-inductive. By making one of the condensers variable, the voltage attenuation can be varied.

The Educational Value of the A.C. Potentiometer.— Not least amongst the uses of the a.c. potentiometer is its employment for the demonstration of many a.c. phenomena and the proof of various circuit theorems. The author has designed a number of pieces of apparatus specially intended to demonstrate fundamental a.c. phenomena of this nature to students. In most cases these demonstrations concern effects which are not easily shown as clearly by other means. The following devices are arranged as experiments for the a.c. potentiometer :

The torque of an eddy current motor.
The complete vector diagram of a transformer on various loads.
The production of a rotating magnetic field.
The relation between mechanical force and electrical energy.
The characteristics of a long transmission line.
The generalized circuit equations of complex circuits.
The attenuation of voltage, current and power in networks and the mutual admittance and impedance relationships.
The measurement of the decibel.
Unique impedances and circuit paradoxes.
The demonstration of motional impedances.[13]
Constant current to constant voltage transformation and vice versa.

There are many other applications capable of giving graphic instruction to the student of a.c. circuit theory and particularly of demonstrating that every electrical machine can be reduced to an equivalent circuit.

One feature of particular interest is the ease with which the phenomenon of homographic transformation [14] in a circuit containing resistance and reactance can be measured. In any circuit containing resistance and reactance in series, parallel or however complex, if the value of one branch changes, the

loci of the vector voltages and currents in the other branches are arcs of circles. This effect is rigidly true so long as the supply voltage is constant and the circuit is composed of "linear" conductors in which the current flowing is proportional to the voltage acting. If either the resistance or the reactance changes or both change simultaneously in a fixed ratio in part of a network, or further the frequency changes, the phenomenon occurs with great precision. Departure from the circle diagram is essentially due to some non-linear impedance in the circuit. The ease with which the vector values of the voltage and current can be measured at different parts of a circuit without disturbing it in any way makes the potentiometer particularly adapted for this purpose. The results can be plotted on squared paper without any calculations and the circle diagrams are at once seen. The phenomenon depends upon the fact that the inverse curve of a straight line in polar co-ordinates with respect to any point not on the line is a circle and the inverse curve of a circle with respect to another centre of inversion is again a circle.[15] Thus if the impedance locus varies in a straight line the corresponding admittance will vary on a circle because admittance is the inversion of impedance. Successive branches of the circuit represent successive circles alternately of admittance and impedance. Since the voltage in any circuit is proportional to the impedance, an impedance circle is also a voltage circle to some other scale. Similarly, because the current in a circuit is proportional to the admittance, an admittance circle is also a current circle to some other scale. The method is particularly useful in showing the performance of networks (among which can be included many machines) under varying load conditions. Many circuit problems can also be solved by means of circle diagrams.[16–18]

NOTES AND BIBLIOGRAPHY.

1 Spooner, Thomas: "Some Applications of the A.C. Potentiometer," *J. Sci. Instrum.*, 1926, iii, 214–21.

2 Spooner, Thomas: "Some Applications of the A.C. Potentiometer," *J. Opt. Soc. Amer.*, 1926, xii, 217–26.

3 Bekku, S., Mayekawa, K., Dote, M.: "On the Field-measurement of the Constants of Overhead Transmission Line with A.C. Potentiometer," *Res. Electrotech. Lab. Minist. Commun. Japan*, 1927, No. 222, p. 46. (In Japanese; short English abstract.)

[4] BOWEN, A. E., and GILKESON, C. L.: "Mutual Impedances of Ground Return Circuits," *Trans. Amer. Inst. Elect. Engrs.*, 1930, xlix, 1370–83.

[5] ARNOLD, A. H. M.: "Precision Testing of Current Transformers," *J. Instn. Elect. Engrs.*, 1930, lxviii, 898–905.

[6] WILLANS, P. W.: "Low Frequency Intervalve Transformers," *J. Instn. Elect. Engrs.*, Vol. 64, No. 358, October, 1926.

[7] SMITH, B. S., and SMITH, F. D.: "An Instrument for the Production of known small High Frequency Alternating Electromotive Forces," *Proc. Phys. Soc.*, 1929, xli, 18–28.

[8] BROUGHTON EDGE, A. B.: "The Principles and Practice of Geophysical Prospecting," Cambridge Univ. Press, p. 50. B. Patent Specfcn. No. 19120/30.

[9] GALL, D. C.: "Some Experiments relating to Geophysical Prospecting," *J. Sci. Instrum.*, viii, No. 10, October, 1931.

[10] JENSEN, AXEL J.: "Potentiometer Arrangement for measuring Micro-voltages at Radio Frequencies," *Phys. Rev.*, 1925, xxvi, 118–19.

[11] MACALPINE, WILLIAM W.: "A Radio-frequency Potentiometer," *Proc. Inst. Radio Engrs.*, 1930, xviii, 1144–54.

[12] ASTBURY, N. F., and IORWERTH JONES, T.: "A Capacitance Attenuator," *J. Sci. Instrum.*, xiii, No. 12, Dec., 1936.

[13] KENNELLY, A. E., and PIERCE, G. W.: "The Impedance of Telephones as affected by Motion of their Diaphragms," *Proc. Am. Acad. Arts and Sc.*, 1912–13, xlviii, 111–51; also *Elec. World*, 1912, lx, 560–5.

KENNELLY, A. E.: "Elec. Vibration Instruments," pp. 73, 74, 76, 99–114 (Macmillan).

[14] BARTLETT, A. C.: "The Theory of Electric Artificial Lines and Filters," pp. 124–42 (Chapman & Hall).

[15] LA COUR and BRAGSTAD: "Theory and Calculation of Electric Currents," pp. 62–8.

[16] GALL, DOUGLAS C.: "Railway Track Circuits," p. 226, 1933 (Pitman, London).

[17] MACALPINE, WILLIAM W.: "The Resistance of Bismuth in Alternating Magnetic Fields," *Phys. Rev.*, 1931, xxxvii, 624–33.

[18] GALL, DOUGLAS C.: "Correction of Frequency Error in Moving Iron Voltmeter," *J. Sci. Instrum.*, ix, No. 11, p. 361, Nov., 1932.

CHAPTER XII.

FACTORS GOVERNING THE CHOICE OF A POTENTIOMETER CIRCUIT.

It will be obvious that there are endless ways in which circuits can be contrived to act as a.c. potentiometers. The choice of a system is conditioned by the ease with which it can be calibrated for its particular purpose and its stability.

All a.c. supplies fluctuate both in voltage and in frequency. If perfectly steady sources were available almost all circuits would be of equal virtue, but fluctuations magnify the defects of some potentiometer systems.

It is of no great practical advantage to be able to standardize the potentiometer with great precision under perfectly steady conditions because there are very few a.c. supplies which are steady to 1 in 1,000 both in magnitude and frequency. It is of far greater practical value to be able to obtain steady readings on a slightly varying supply, and to know that these readings refer to some steady value of the supply. This is quite feasible in practice, so that with a suitably arranged circuit an accuracy superior to the steadiness of the supply can be attained; the results obtained being those values which would occur in the circuit if the supply were steady at the value at which the potentiometer reads correctly. This may be called the theoretical test voltage. Under normal conditions of use the pointer of any accurate current-measuring device will be continually swinging. Its precision is only helpful in the preliminary setting up of the potentiometer in order to determine the theoretical testing voltage with accuracy.

It is possible to stabilize the voltage supply to the a.c. potentiometer so that it is unaffected by the fluctuations, utilizing a thermal bridge circuit of positive and negative temperature coefficient arms, and thus give steady values of poten-

tial accurate in absolute value. It is an interesting experience to use such an a.c. potentiometer, because the results are extremely bad. The balance is so unsteady that only very approximate readings can be obtained, because of the fluctuations in the circuit under test, which cannot be so stabilized, if any appreciable power is required.

If the voltage on the potentiometer varies in exact step with the variation of the voltage on the circuit under test, the potentiometer will be much easier and more accurate to use than one in which the above relationship does not hold. If the potentiometer could vary its voltage with frequency fluctuations in exact step with the voltage variations of the circuit under test, it would be a great convenience for obtaining steady readings. Unfortunately this can only occur to a limited extent. The effect of frequency fluctuations is to change the volt drop on the reactive parts of the circuit. If the potentiometer circuit contains a large reactance, the phase of the current and the magnitude of the volt drop will swing about with frequency fluctuations. It may so happen that the circuit under test will behave in a similar way, but the conditions for this compensation to hold accurately are very restricted.

These effects can be minimized by employing the inphase potentiometer closely coupled to the supply, so that the voltage in that circuit is always proportional to the supply voltage. If the circuit under test contains series inductances, the inductances can be retained in the inphase potentiometer circuit. The quadrature voltage of the potentiometer will always vary with frequency if supplied by reactive means such as a mutual inductance or a condenser. If the circuit under test contains series inductances, then either mutual inductance or series capacitance in the quadrature potentiometer will provide a degree of compensation, but series inductances will give aggravation. If, however, the circuit under test contains series capacitance, the series inductance is necessary in the quadrature potentiometer. In the author's form of co-ordinate potentiometer the inductance can be transferred or omitted from either circuit when desirable.

In the Larsen and Pedersen circuits the phase and magnitude of both components swing with frequency fluctuation.

If it is desirable to omit all inductance from both inphase and quadrature potentiometers, the standardization of the circuit can be carried out by reference to a circuit across the

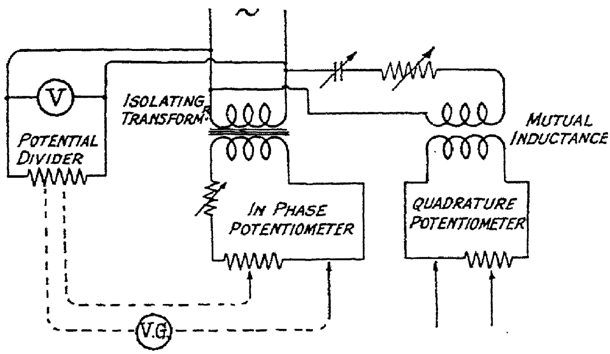


FIG. 104.—Voltage standardization of a.c. potentiometer.

supply. This scheme has several advantages at the higher frequencies. Standardization by voltage instead of current is indicated in Fig. 104.

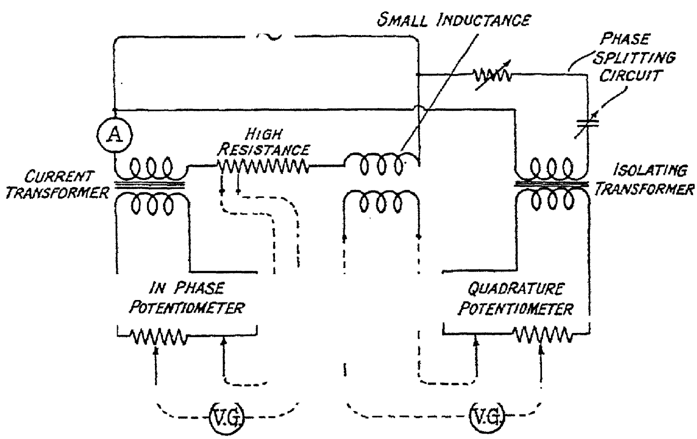


FIG. 105.—Voltage standardization of a.c. potentiometer.

A more detailed circuit for standardizing both potentiometers is shown in Fig. 105. The inphase potentiometer is standardized against the resistance divider and the quadrature

potentiometer against the mutual inductance. The value of the resistance divider is sufficiently high to make the time-constant quite small, so that the inphase potentiometer does not swing with frequency. This form of the circuit is often best associated with the measurement of circuits of small reactance. In this case the quadrature components are small and the volt drop on the quadrature potentiometer can be made a small fraction of that on the inphase potentiometer. This also means that the value of the inductance in the standardizing circuit can be kept very small. It is in the flexibility of the circuit arrangements that the virtue of the co-ordinate potentiometer resides. There are many methods of arranging the circuit applicable to special problems. A number of these are described in the patent specification No. 334131.

The presence of large inductances near the circuits under test is always liable to cause stray fields. For this reason condensers are preferable to inductances for producing the quadrature voltages in the potentiometer circuit. In measuring small voltages where values of the order of microvolts have to be observed, the potentiometer must have a very low impedance, otherwise much sensitivity is lost. This is where the low-resistance potentiometer is at an advantage as compared with one using inductances or capacitances in the galvanometer circuit.

Difficulties Encountered in Precise Measurement by means of the A.C. Potentiometer.—One of the most common causes of inaccuracies in a.c. potentiometer measurements is due to induced e.m.f.s. Induced e.m.f.s in the potential leads between the potentiometer and the point of the circuit at which voltage is being measured, or in the galvanometer leads between the potentiometer and the galvanometer, or into the potentiometer circuit itself, or even into the apparatus under test, is equivalent to an additional e.m.f. acting in the circuit which is not a part of the voltage which it is desired to determine, nor a known part of the voltage which is used to make that determination. The effect of such induced e.m.f.s is to give a condition of balance which is untrue, inasmuch as the reading of the potentiometer does not represent the voltage which it is desired to determine, but represents some different voltage

which is the sum of the required e.m.f. and the parasitic induced e.m.f. As already stated, the presence of parasitic e.m.f.s can in most cases be readily determined by short-circuiting the potential leads at the point at which they are connected to the circuit under test. Only if the circuit is free from parasitic e.m.f.s will the galvanometer balance with the potentiometer exactly at zero. If the galvanometer balances with some small reading upon the potentiometer, then it is evident that there is an e.m.f. acting in the circuit which has to be neutralized, or balanced, by this setting of the potentiometer dials. It is generally preferable to endeavour to eliminate these extraneous e.m.f.s, rather than treat them as corrections to the observed readings. They are best eliminated by carefully twinning all the leads which form the potential circuit and the galvanometer circuit. In this way, electromagnetic fields will produce balanced e.m.f.s in the circuit. Occasionally, it is impossible to carry out this procedure, for example, when it is necessary to measure a voltage between two points at some considerable distance apart so that the potential leads cannot be twinned up to that point but must be widely spaced as the two points to which their respective ends are to be attached are approached. Such cases may sometimes introduce great difficulty in finding a proper value for the stray e.m.f. induced in the potential leads, as any attempt which is made to measure it may involve an alteration in the configuration of the actual current circuit which is responsible. It is also important that there should be no direct magnetic action upon the current-measuring instrument or dynamometer in the potentiometer circuit due to stray alternating current fields from some external source. Such interference can best be observed by reversals of the current. If the dynamometer deflexion is being influenced by an external field, reversing the direction of the current through the dynamometer will modify the deflexion of the instrument because its own field will be reversed, while the external field remains unchanged in direction. Where a standardizing mutual inductance is used in the potentiometer circuit, this too must be examined to ensure that it is not the source of an extraneous e.m.f. induced into its winding from an external magnetic field. This is most easily done by disconnecting it from the circuit

and connecting the vibration galvanometer to its secondary winding when any induced e.m.f. will immediately cause a deflexion upon the galvanometer scale. By orientating the mutual inductance, a position can be found where the induced e.m.f. is zero. Stray fields will be greatly reduced if all current-carrying circuits are also twinned, and such pieces of apparatus as transformers and machines are kept as far away as is possible. It is always advisable to make a check of any voltage balance which has been obtained, with the potential leads and galvanometer leads successively reversed. It is in setting up the a.c. potentiometer for precise measurements that care should be taken concerning these points. In this way the presence of parasitic e.m.f.s may be detected before a long series of readings is undertaken.

Thus when all parasitic e.m.f.s have been eliminated, it should be possible to reverse any pair of leads either in the galvanometer circuit, in the potential circuit, in the current circuit feeding the potentiometer, or in the dynamometer circuit and to obtain exactly the same balance. The signs of the potentiometer readings may be reversed, but their values should remain unchanged. If this degree of perfection cannot be obtained then it is advisable to take the mean of the reversed readings rather than subtract the measured stray potential from a single reading, although both methods should give the same results. One of the most fruitful sources of error in refined testing has been due to the bad wave-form of the supply. Many of the older testing generators used for calibration of instruments have very bad wave-forms judged by modern standards, and it is really illogical to use such machines for precise calibration work because ambiguities are bound to occur in the interpretation of these calibrations. As has already been pointed out, under suitably arranged conditions, small fluctuations in the voltage do not necessarily introduce serious inaccuracies in the measurement ; since the circuits are compensatory, the circuit under test varying in the same way as the potentiometer on which measurements are made. Frequency fluctuations constitute, however, a source of variation and this will naturally be expected in any reactive measurement, because no fixed value can be assigned to the reactance being measured

unless the frequency remains constant. One of the most successful means adopted for supplying the a.c. potentiometer and the apparatus under calibration is to use a smooth core rotary converter excited from batteries of ample capacity, the former being mechanically coupled to a synchronous motor fed from the frequency controlled National Grid system. In this way a very steady frequency can be obtained. In addition, the output voltage of the rotary converter is very steady because it depends upon the voltage of the battery excitation. If the combination is further arranged with separate current and voltage machines, the phase of which can be altered with reference to each other, a very wide range of precise measurement can be covered, since voltage measurements can be carried out by means of the voltage machine and current measurements where large values at low voltages are involved by means of the current machine. The great advantage of being able to adjust the phase of either of these machines enables standardization to be made at various power factors and always under the most favourable phase relationship conditions with regard to the potentiometer. This is the scheme adopted at the Royal Technical College, Glasgow.

Another source of difficulty in connexion with precise potentiometry arises from stray currents entering the galvanometer circuit owing either to the capacitance or to low insulation between the potentiometer circuit and some high-voltage source. Errors from this source can be very serious, because very small currents entering into high-impedance circuits may cause relatively large volt drops. If the simple case is considered of a current entering the potentiometer circuit from the supply mains on the primary side of the isolating transformer, this effect will be readily understood. Such a current would find its way into the potentiometer and pass through both potential leads back to the circuit under test and so back to the opposite mains.

The galvanometer would be in one of the potential leads and a volt drop would occur across the galvanometer equal to the stray current passing through it (or half the total stray current if it was divided equally between the two potential leads) multiplied by the impedance of the galvanometer. There

would be no compensating volt drop in the opposite lead, and thus the volt drop across the galvanometer would have to be compensated by an equal and opposite e.m.f. derived from the potentiometer by a false setting. The reading might be further vitiated by any similar volt drop occurring in a circuit of high resistance under test, depending upon the actual paths taken by the stray currents. It is easy to see that in cases involving a high-impedance galvanometer, or a high-impedance test circuit, the magnitude of these volt drops could be relatively large, that is, of the order of hundreds of millivolts even with quite small leakage currents. It is essential, therefore, that no such leakage current should be allowed to enter the potentiometer. This is achieved by the use of screens between the primary and secondary windings of the isolating transformer, these screens being carefully earthed or connected to one side of the supply voltage, in order to preclude the entry of leakage or capacitive currents into the potentiometer. When working at higher frequencies, it is also necessary to make use of the screens provided round the potentiometer circuit itself, so that the capacitances of these circuits cannot cause stray currents of this nature to enter. Occasionally even these precautions are insufficient; such a case occurs when the point at which the potential leads are connected to the apparatus under test are at relatively high voltage above earth or above the potentiometer source of supply. A particular example of this arises in the measurement of earth impedances in connexion with geophysical prospecting described in Chapter XI. In this class of measurement, it is essential to measure the volt drop upon the surface of the ground, and although the actual volt drop measured between the two potential points may be not more than 1 volt, the surface of the earth at which these measurements are made may be 100 or more volts above the potential of the isolating transformer supplying the potentiometer. The result is that capacitive currents flow freely back from the potential leads through the isolating transformer, and the voltage measurement obtained is useless without some method of compensation. The method employed [1] is to introduce a balanced isolating transformer into the potential leads. This transformer is differentially wound and the detector, whether

a galvanometer or a telephone, is connected to a completely isolated secondary winding, as shown in Fig. 61.

Dr. C. V. Drysdale has proposed the use of a differential vibration galvanometer for this purpose.[2] This instrument, however, is in the author's experience much more difficult to construct than the transformer.

Adjustments for Variation in Supply Voltage.—It is of the greatest importance that no change be made in the potentiometer circuit during a series of readings and on no account should the potentiometer current be adjusted by the rheostat embodied in the potentiometer, as by so doing the phase-splitting between the potentiometers will be changed. Adjustment for variation in the supply voltage should always be provided externally to the potentiometer as indicated in Figs. 62 and 63.

When it is necessary to adjust the controlling rheostats to compensate for slight variations of supply voltage the phase relationship between the potentiometer voltage system and the circuit under test is altered. In the calibration of an ammeter or voltmeter or in any other measurement where only the magnitude of a voltage vector is required, this is not important provided that the standardization of the potentiometer is checked after any adjustment has been made. For a wattmeter calibration or any measurement involving the determination of two or more quantities, which must be related to each other in both magnitude and phase, it is essential that all the relevant measurements be made without adjustment of any of the controlling rheostats. If the variation of supply voltage is appreciable during any one set of readings it becomes necessary to take several sets of readings and average them to obtain the maximum possible accuracy.

It may be found that it takes as much as two or three hours before the machine supplying the potentiometer and test circuit reaches sufficient equilibrium to provide the stability of conditions which is essential for the highest precision of which the potentiometer is capable.

It is convenient to have a voltmeter across the supply on the test side of the controlling rheostats so that the voltage can be readjusted to the same conditions if any variation occurs.

In this way the relative voltages of the potentiometer and the circuit under test remain undisturbed.

In obtaining a series of readings, as in the case of a network of which one element is being varied, it is essential that no adjustment should be made to the potentiometer circuit during the series of readings, and the control must therefore be external as described above or the readings will not co-relate. If the variation to the network alters the load so that the voltage varies, this is not very serious so long as the voltage on the network and on the potentiometer remain equal. The readings will co-relate because they will retain their proportionality. When necessary the voltage can be readjusted to its original value by the control rheostat on the supply side of the test circuit and the potentiometer.

Method of Constructing, Adjusting and Calibrating Resistance Potentiometers.—The coils used in both d.c. and a.c. potentiometers are basically the same, and the same method of construction is employed except that the type of winding used in the a.c. potentiometer will preferably be such that the residual inductance is small, while the internal leads should be so disposed that the potential circuits are as nearly non-inductive as possible in order to reduce the effect of magnetic fields.

The potentiometer coils should be wound with manganin[3-4] wire to reduce thermo-electric effects. This is important during the process of adjusting the coils as well as when using the potentiometer on direct currents. It is very difficult to adjust accurately a resistance coil with large thermo-electric effects present, as the process of handling or scraping the coil affects the volt drop on the coil by which its resistance is measured. In the same way, resistance coils with large temperature coefficients are very difficult to adjust accurately because their resistance varies continually while handling.

Potentiometer coils for a precise instrument, unless their resistance be less than one ohm, should be wound on metal bobbins insulated with a layer of shellacked silk. The manganin wire should be silk-covered and wound evenly in a bifilar loop upon the bobbin. Undue tension should not be employed in winding the wire or the mechanical stresses will cause the

resistance value to change. The time-constant of a bifilarly wound coil of wire not larger than No. 34 gauge and not more than 100 ohms in resistance, will not exceed 10^{-6} second. For some a.c. potentiometer coils it may be advisable to use a still more non-inductive type of winding, such as a flat mica card with two windings in reversed directions, or a fine twisted bifilar spiral, but no type of specially non-inductive winding gives such good resistance permanence as the smoothly wound cylindrical coil on a metal bobbin. This appears to be due to the mechanical stresses introduced in the more complicated types of winding.[5] So long as the time-constant of every coil is the same, a small residual inductance is not serious in the case of coils for a.c. potentiometers, while the permanence of resistance is a matter of the highest importance. After the coils are wound they must be heat-treated to relieve stresses. Annealing at 550° C. is really required, but this is out of the question with silk insulation, and the coils should be baked at about 140° C. for a period of from 60 to 80 hours. If the initial stresses are not too great, this treatment is usually adequate. In the case of open wound spirals of bare manganin wire, as used in some high precision d.c. potentiometers, the heat-treatment can be made at 550° C. Unless a reducing atmosphere is used, scale forms on the wire. This must be removed by pickling, and the resistance wire must not be bent after annealing. After baking, the coil resistance should be measured, the surplus cut off and copper end wires silver-soldered to the ends of the manganin wires at the appropriate points. A slightly larger size of copper wire should be used, but not much larger, or its weight will tend to break the manganin. The silver-soldered joint must be very carefully cleaned of fused flux or the coil will corrode later. The coil is usually treated with one or more layers of shellac varnish to protect it from atmospheric effects, although it is known that shellac varnish causes variation in the coil resistance.[6-7] It is a matter of choosing the lesser evil; unvarnished, the manganin will be attacked atmospherically (unless hermetically sealed), and this is a more serious matter than the varnish effects.

Having been wound, heat-treated and fitted with copper ends, the resistance of the coil should be adjusted to its nominal

value. A Wheatstone bridge is used for this purpose and the coil adjusted by scraping the manganin while the copper ends are held in the bridge terminals until the coil value is correct within say 0·02 per cent. low of its nominal value. The precision of adjustment at this stage will depend upon the resistance of the coil and the ultimate degree of precision aimed at. It is useless to adjust very low resistance coils to great precision at this stage because the effect of soldering them into their final assembly may greatly change their values.

With few exceptions, among which is the shunted dial type of resistance variation, potentiometer dials consist of a set of coils upon which the volt drops should be exactly equal.[8] The method of adjustment is, therefore, by the volt drop upon the coils. When the coils have been assembled into the potentiometer, current is passed through them. Another potentiometer is then used to compare the volt drop upon the coils forming the dial under test. A special pair of fingers is usually required to enable the volt drop to be measured upon successive coils. A preliminary test of the volt drops is made upon all coils and the coil having the highest resistance is selected to form the standard. The volt drop on all the other coils is then brought up to agree with this one by gently scraping the manganin wire near the copper ends. Great care is required in the process, and the approach to the correct volt-drop value must be made very gradually, otherwise it will be overshot. The coils do not settle down to their final value immediately after scraping, and it is necessary to go round the dial several times if a high degree of precision is desired. The second potentiometer used for comparing the volt drops upon the coils under adjustment must be capable of a very fine control of potential, and a precision instrument should be employed although the actual voltage value is of no great importance. It is essential, too, that the current in both circuits is extremely steady, and, in practice, large-capacity accumulators are connected to both circuits several hours before the process of adjustment, to allow the currents to settle down. By this method the coil resistance can be adjusted to equality within 1 part in a million by a skilful person, but the final values to which the coils settle down will not be better than 1 part in 100,000 of equality.

Other effects, such as contact variation, temperature gradients and stress changes will combine to reduce the effective equality still further.[7] A dial of eighteen coils, where the difference of effective equality between any two coils does not exceed 1 in 20,000, can be regarded as having very good adjustment. In terms of ten coils, this will represent an error in mean resistance gradient of only 1 in 200,000. The relative volt drops upon successive dials of the potentiometer are adjusted in the same way; the comparison of ten steps of a lower decade being adjusted to be exactly equal to one step of the dial above. Slide wires are calibrated in a similar manner, using the gradient of ten equal resistance coils in the second potentiometer to calibrate ten points on the slide-wire scale. The voltage range of the second potentiometer is adjusted for each test, so that advantage can be taken of the largest number of figures of which the second potentiometer dials are capable. If two potentiometers are constructed simultaneously, one can be used alternately to calibrate the other, since there is need only to be able to control the comparing voltage to a fine degree in the first adjustment of the coils to equality. In practice, it is, however, more convenient to have an accurately adjusted potentiometer for the second instrument.

The final calibration of the potentiometer is most easily made by comparing the equality of successive steps on the first dial against the total of the second dial, using the second potentiometer to balance the first at any required setting. For example, with the first dial at 0 and the second at its total setting, the potential should be balanced by means of the second potentiometer. When this is accurately balanced, set the first dial to 1 and the second dial to 0. The balance should be undisturbed, but if sufficiently sensitively measured, there will always be a small difference. This should be rebalanced and the difference represents the error on the 0–1 stud of the first dial in terms of the second dial. Now, leaving the first dial at 1, turn the second dial to its full value. Rebalance on the second potentiometer and repeat the process, thus determining the error of the 1–2 stud of the first dial in terms of the second dial. This process of dial substitution can be carried out with any number of dials or with a slide wire, starting

preferably from the lowest. The errors so determined can be tabulated as corrections to be applied for work of the highest precision. Thus, the accuracy of a potentiometer can always be determined if another one is available.[8] This is one advantage of having two potentiometers embodied in the co-ordinate a.c. potentiometer. The coils of a.c. potentiometers are adjusted on d.c., since it would be impossible to obtain sufficiently steady conditions with an a.c. supply.

It will be clear that all a.c. measurements depend upon d.c. standardization in the first place. The definition of a sine wave of alternating current of 1 ampere is essentially a current which produces the same mean magnetic field as 1 ampere of direct current. The mean force of each magnetic field will be the same.

One a.c. volt produces the same mean electrostatic field as a d.c. volt, and therefore the same mean force. In both cases, the alternating force is varying with double frequency between zero and twice the d.c. value which is the mean.

The definition of heat equivalence between a.c. and d.c. is really a power equivalence and not a current measurement. The heat in a measuring device is due to the power, which will be I^2r. If the heat is the same with a.c. as with d.c., the current will be the same if r has remained unchanged, but the heat is just as much due to the volt drop Ir as to the current.

In general, the equivalence of the magnetic field is the most convenient link between a.c. and d.c. at the low voltages used in measurement. This necessitates a device in which a.c. and d.c. produce the same mean magnetic fields (*i.e.*, there is no frequency effect). This is logically the iron-free dynamometer.

The electrostatic voltmeter is preferable to the electromagnetic dynamometer from the view-point of its purity, but the forces available at low voltages are so small that it is essentially a very delicate instrument of restricted utility and is furthermore unportable. When voltage equivalence between a.c. and d.c. is established by means of the electrostatic voltmeter, current equivalence can be established by means of the volt drop upon a non-inductive resistance.

It is inevitable, therefore, that the resistance potentiometer in some form or other provides the link by which all a.c. stan-

dardizations are built up from equal mean forces of the electric field.

Care and Maintenance of Precision Potentiometers.—Under no circumstances should the potentiometer resistance coils be handled, nor should their ends be bent, since the least bending of the wire will alter its resistance. The switch contacts require cleaning periodically, say two or three times a year, depending upon the atmospheric conditions in which the instrument is used. It is inadvisable to use abrasive materials upon the contact surfaces; if these are of silver-gold alloy, brass, copper or phosphor bronze, once the surface becomes charged with abrasive, cutting will continue for years, and the abrasive cannot be removed without scraping off the surface of the metal. Contact surfaces charged with abrasive give very variable contact resistances. To clean the switch contacts, it is usually sufficient to squirt a little petrol on to the contact surfaces and operate the switches several times. The dirty liquid and dust should then be wiped away, preferably using a piece of chamois leather. This process may have to be repeated several times if the instrument is very dirty, but very little petrol should be used and none allowed to get on the ebonite panel. When all dirt has been removed, a very little medicinal paraffin should be applied to the contact surfaces. The ebonite or hard rubber panel is best cleaned with distilled water carefully used on a mop so that none gets into the instrument, and finally cleaned off with turpentine. If the panel has not been exposed to light or is made of "loaded" ebonite in which there is no free sulphur, the distilled water treatment is unnecessary, since the function of the distilled water is to remove the sulphuric acid which forms on the surface of ebonite exposed to light.

The most fruitful source of trouble in potentiometers is the fine adjustment rheostat, and this is invariably due to the difficulty of making good sliding electrical contact on a thin wire. If too much pressure is used, the friction causes mechanical trouble by "firing" the surface, while if the pressure is insufficient the contact is variable. There is a certain pressure with suitable materials which produces a burnish. These are the best conditions. Again, no abrasive should be used in

cleaning if the design is correct. Petrol and medicinal paraffin are the best cleaning agents, as in the case of the other contacts. Dust will always collect on the contact surfaces and if there is nowhere for it to get away, it will gradually accumulate under the moving contact surface. A few small channels in the contact into which the dust can escape will prolong the time between cleaning.

Slide wires are usually made of platinum-silver alloy. Some specimens exhibit freak behaviour for which no satisfactory explanation has been found. Such a wire will give continual trouble, a film of high resistance forming on its surface in about 24 hours. The film can be readily removed by rubbing the wire with a piece of ordinary writing-paper, but it cannot always be removed by the metal slider of the instrument itself even when operated many times. In such cases the film is a source of intermittent contact and the only satisfactory cure is a new slide wire of better material. The wear on a platinum-silver slide wire in normal use, if uncharged with abrasive, is very small indeed and the resistance thereof does not change appreciably with years of use. It is the slider which wears. Some wires have "drawing ripples" on their surfaces, and these always give excessive wear to the slider and generally make poor contact.

NOTES AND BIBLIOGRAPHY.

[1] Gall, Douglas C.: "Some Experiments relating to Geophysical Prospecting," *J. Sci. Instrum.*, 1931, viii, 305–13.

[2] Drysdale, C. V.: "Progress in the Design . . . of Electrical Instruments," *J. Sci. Instrum.*, March–June, 1927, iv, Nos. 6–9.

[3] St. Lindeck: "Alloys for Resistance Coils," *The Electrician*, Vol. 30, 1893, p. 119.

[4] St. Lindeck: "Über die Haltbarkeit von Kleinen Widerstande am Manganene Bleck im Praktischen Gebrauch," *Zeit. für Instrumentenkunde*, Vol. 23, 1903, p. 1.

[5] Curtis, H. L., and Grover, F. W.: "Resistance Coils for A.C. Work," *Bull. U.S. Bur. Stand.*, Vol. 8, No. 3, 1911.

[6] Rosa, E. B., and Babcock, H. D.: "The Variation of Resistances with Atmospheric Humidity," *Bull. U.S. Bur. Stand.*, Vol. 4, 1907–8, p. 121.

[7] Rayner, E. H.: "The Effect of Design in the Stability of Manganin Resistance," *J. Sci. Instrum.*, xii, No. 9, 1935, p. 294.

[8] Gall, Douglas C.: "Winding and Adjusting Resistance Coils," *J. Sci. Instrum.*, May, 1934, xi, No. 5.

CHAPTER XIII.

THE REPRESENTATION OF ALTERNATING CURRENTS BY COMPLEX QUANTITIES.

The use of complex quantities as a means of expressing alternating currents and voltages is very convenient. It permits the calculations involving these quantities with great simplicity and without the use of trigonometrical tables.

The complex quantity, consisting of a real component and a quadrature component, is primarily an algebraical expression which lends itself very aptly to the representation of sine waves of current and voltage because in any circuit there are two effects.

There is the effect which is proportional to the current or voltage such as the volt drop in, or current through, a resistance. This effect is in phase with its cause.

The second effect is proportional to the rate of change of the current or voltage, such as the volt drop on an inductance or capacitance. This effect is 90° out of phase with its cause because the rate of change of a sine wave is a cosine wave which is simply a 90° phase displacement of the sine wave.

Since the laws governing the current and voltage in any circuit depend upon these two effects only, the representation in two components at right angles which results from these laws would seem more basically true than the polar representation which essentially combines the effects of direct proportionality and rate of change effects.

Representation by complex quantity is, however, not without its dangers, and the student must never lose sight of what the complex quantity really represents. A complex quantity used to represent a current implies two alternating currents displaced 90° in time phase with respect to each other. So far as the behaviour of the circuit is concerned, the currents can

be treated as though they existed independently and then superposed. This is the superposition theorem. Exactly the same applies to a voltage represented by a complex quantity.

When the product of the current and the voltage are required, very careful consideration must be given to the exact meaning of the symbols. If the current is represented by the complex quantity $a + jb$ and the voltage by $c + jd$, both a and c will be sine waves of current and voltage respectively in phase with each other. The power which is the product of current and voltage will obviously be ac. In exactly the same way the quadrature component of current b is in phase with the quadrature component of voltage d, so that a further component of power will be present equal to bd. The total power will therefore be $ac + bd$. It will be evident that with different signs the products will follow the usual algebraic laws, the products of the components being negative if one sign is negative, but positive if both are negative. The same reasoning can be applied to the quadrature products. The product of that component of the voltage which is in quadrature with the current represents reactive volt-amperes.

Reactive volt-amperes represents a storage of energy because it is really the product of the current and the flux it produces. With an alternating current the flux induces a voltage which lags behind it by 90°, because the induced voltage is proportional to the rate of change of the flux. Thus the induced voltage due to the flux is 90° behind the current producing the flux. The product of the current and the component of voltage lagging 90° behind it gives, therefore, the product of the current and flux and is the energy stored in the flux. The flux is alternating like the current which produces it and is in phase with it. The product is therefore a $(\text{sine})^2$ function, that is a double-frequency sine wave of all positive values, rising from zero to twice its mean height. Its mean amplitude is the average stored energy. The product of current and reactive volts is in reactive volt-amperes which can be converted into joules by dividing by $4\pi f$.

When two complex quantities are used to represent the current and voltage the components represent separate sine waves at 90° displacement. The component of current a is in

phase with the component of flux which induces the voltage lagging 90° behind it. This is the component d. Therefore energy is stored by the product ad. The component of flux which induces the voltage c will be 90° in advance of c, that is 180° out of phase with the current component b. The energy stored by this product will therefore be negative with respect to the first product. That is, the reactive volt-amperes will be $ad - bc$. It should be noted that neither the power nor the reactive volt-amperes is the algebraic product. The algebraical product of the complex quantities does not give the power and reactive volt-amperes. The reason lies in the change from a single-frequency quantity to a double-frequency quantity. The student is often exasperated by this paradox and its apparent dangers. In experimental work there is less chance of confusion than in analytical work.

The representation is a mathematical convenience and must be kept under control. The simplest way to avoid all ambiguity in circuit calculations is to work in either voltage or current, but not in both. Thus if the calculation is carried out in current, the voltages will be expressed as current times impedance. If current is kept as a purely numerical quantity and impedance and admittance as complex quantities, all circuit calculations, no matter how complicated, can be carried out by the ordinary algebraic laws of complex quantities. The advantage of considering either current or voltage as a numerical quantity acting in a complex circuit, can be seen from the following example. Consider the case of a circuit shown in Fig. 106.

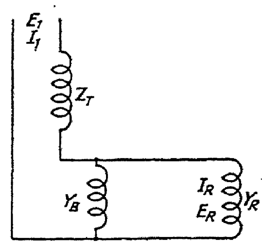


FIG. 106.

A supply voltage E_1 feeds a relay of admittance Y_R over a circuit consisting of a series impedance Z_T and a shunt circuit of admittance Y_B.

The voltage at the relay is E_R and the current I_R. Both are numerical values.

It will be evident that the total current will be given by

$$I_1 = E_R(Y_R + Y_B).$$

The volt drop on the series impedance will be

$$I_1 Z_T = E_R(Y_R + Y_B)Z_T.$$

The supply voltage will therefore be

$$E_1 = E_R\{1 + Z_T(Y_R + Y_B)\}.$$

Substituting the complex values of Z_T, Y_R and Y_B gives

$$E_1 = E_R[1 + (r_T + jx_T)\{g_R + g_B + j(b_R + b_B)\}]$$

$$E_1 = E_R[1 + r_T(g_R + g_B) - x_T(b_R + b_B) + j\{x_T(g_R + g_B) + r_T(b_R + b_B)\}].$$

The term in the bracket has been expanded by algebraic multiplication of the complex quantities in which $j^2 = -1$. If E_R is a numerical quantity, E_1 will be given in its vector components in relation thereto. The power input given by E_1 and I_1 can readily be derived as follows.

In terms of E_R,

$$\begin{aligned} I_1 &= E_R(Y_R + Y_B) \\ &= E_R\{g_R + g_B + j(b_R + b_B)\}. \end{aligned}$$

This complex expression gives the vector relationship between I_1 and E_R. The power will be given by the sum of the products of the components of voltage and current which are in phase with each other. Taking these products in the case of E_1 and I_1 gives the power

$$P_1 = E_R^2[(g_R + g_B)\{1 + r_T(g_R + g_B) - x_T(b_R + b_B)\} + (b_R + b_B)\{X_T(g_R + g_B) + r_T(b_R + b_B)\}] \text{ watts}$$

where E_R is the numerical value of the voltage. This is not the algebraic product of the two complex quantities representing E_1 and I_1.

Similarly the reactive volt-amperes will be given by

$$Pj = E_R^2[(g_R + g_B)\{x_T(g_R + g_B) + r_T(b_R + b_B)\} - (b_R + b_B)\{1 + r_T(g_R + g_B) - x_T(b_R + b_B)\}].$$

Treating E_R as a numerical value is really the same as fixing its phase at zero or fixing the co-ordinate system.

It is evident from this example that it is most convenient for calculation purposes to regard all voltages or currents as numerical quantities until their vector relationships are defined by the complex quantity which represents the circuit impe-

dances. In this way a vector voltage or current always appears as the product of a numerical value operated upon by a complex quantity. Thus all voltages and currents are symbolically numerical quantities and only the relations between them complex. For this reason no indication is given in the symbols used in the text of whether a measured voltage or current is a vector or a simple numerical value. If it is complex, this will at once manifest itself either in the practical measurements or in the analytical calculations. The introduction of vector values of both current and voltage into a formula is only justifiable when the co-ordinate system of each is the same. In a.c. potentiometer measurements this is a basic implication. In analytical work, the complex relationship can only be inferred from the circuit values, not from the current and voltage values in themselves.

The Calculation of Results from Co-ordinate Measurements.—When the current and voltage acting in a circuit are measured by means of the co-ordinate a.c. potentiometer, the results appear in vector form $E = \pm a \pm jb$ where a is the inphase component and b is the quadrature component. Currents are invariably measured by the volt drop on a non-inductive resistor r so that $I = 1/r(\pm a \pm jb)$. These vectors can be plotted at once upon squared paper.

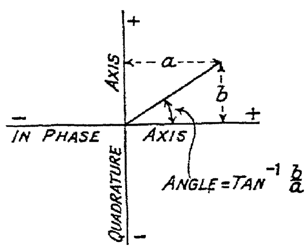


FIG. 107.

If the horizontal axis is taken as the inphase and the vertical axis as the quadrature, the vector appears as in Fig. 107, its length will be $\sqrt{a^2 + b^2}$ and the phase angle with respect to the reference axis will be $\tan^{-1} b/a$.

When the current flowing in a circuit and the voltage are both measured in this form, all the electrical properties of the circuit can be readily calculated. A good example is to consider the circuit shown in Fig. 108, consisting of a choke or iron-cored inductance in series with a non-inductive resistor. The choke is provided with a secondary winding, the open circuit voltage of which can be measured. The four voltages

re measured; E_1 at the terminals, E_2 across the non-inductive esistor, E_3 across the choke and E_4 across the secondary winding. Let $E_1 = a_1 + jb_1$, $E_2 = a_2 + jb_2$, etc., assuming all signs

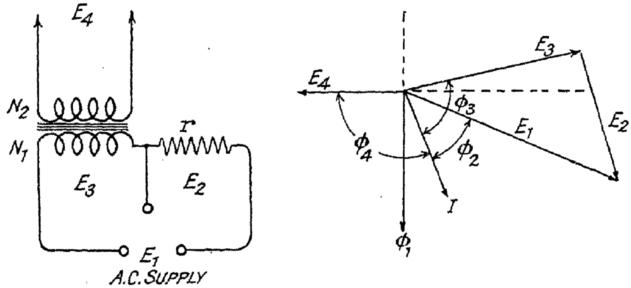


FIG. 108.—Measurements on an inductive circuit.

positive until actual substitution of measured values is made. It is obvious that $E_1 = E_2 + E_3$.

That is $$a_1 = a_2 + a_3$$
and $$b_1 = b_2 + b_3.$$

In making such a set of measurements, this check upon the sum of the components should always be made. It is at the same time an easy check upon the number of figures to which the readings are reliable. The current flowing in the circuit will be

$$I = 1/r(a_2 + jb_2) \text{ amperes.}$$

The power dissipated in any part of the circuit will be the sum of the products of the components of current and voltage in phase with each other. Thus the total power delivered to the circuit will be

$$P_1 = (a_1a_2 + b_1b_2)1/r \text{ watts.}$$

The power in the resistance will be

$$P_2 = (a_2^2 + b_2^2)1/r \text{ watts.}$$

The power in the choke will be

$$P_3 = (a_2a_3 + b_2b_3)1/r \text{ watts.}$$

The energy stored in any part of the circuit will be the difference between the product of the components of current and

voltage in quadrature with each other. There will be no energy stored in the resistance because

$$P_{j2} = (a_1 b_1 - a_1 b_1)1/r = 0.$$

The energy stored in the choke will be

$$P_{j3} = (a_2 b_3 - a_3 b_2)1/r \text{ reactive volt-amperes.}$$

This will be the same as the total stored energy

$$P_{j1} = (a_2 b_1 - a_1 b_2)1/r$$

because $a_1 = a_2 + a_3$ and $b_1 = b_2 + b_3$.

The impedance of any circuit will be given by the voltage divided by the current. Similarly, the admittance will be given by the current divided by the voltage.

The impedance of the choke will be

$$Z_3 = r\left(\frac{a_3 + jb_3}{a_2 + jb_2}\right) \text{ ohms.}$$

Rationalizing by multiplying numerator and denominator by $a_2 - jb_2$ gives

$$Z_3 = r\left(\frac{a_2 a_3 + b_2 b_3}{a_2^2 + b_2^2} + j\,\frac{a_2 b_3 - a_3 b_2}{a_2^2 + b_2^2}\right) \text{ ohms.}$$

That is the impedance consisting of two components, the resistance and reactance respectively is

$$Z_3 = r_3 + jx_3$$

where the resistance

$$r_3 = r\frac{a_2 a_3 + b_2 b_3}{a_2^2 + b_2^2} \text{ ohms.}$$

And the reactance

$$x_3 = r\frac{a_2 b_3 - a_3 b_2}{a_2^2 + b_2^2} \text{ ohms.}$$

The relation between these components and the power and stored energy is obvious. The effective resistance of any circuit is the power dissipated in it divided by the current squared, and the effective reactance is the stored energy divided by the current squared.

It should be noted that r_3 is the effective resistance of the choke, not the d.c. resistance.

The effective inductance L_3 of the choke with current I

owing will be derived from the reactance by dividing by $2\pi f$ here f is the frequency.

The inductance of a circuit is defined as flux turns per mpere. That is

$$\frac{\Phi N}{I} = L = \frac{x_3}{2\pi f}$$

where Φ is the r.m.s. flux.

If N_1 is the number of primary turns then flux would appear o be given by

$$\Phi = \frac{x_3}{2\pi f N_1} \cdot I.$$

This is not so in the case of an iron-cored circuit and is a well-known pitfall in measuring a.c. flux.[4]

The flux in the iron must be derived from the induced secondary voltage E_4.

By the well-known relationship

$$\Phi' = \frac{E_4}{\sqrt{2}\pi f N_2} \cdot 10^8 \text{ gauss.}$$

That is

$$\Phi' = \frac{\sqrt{a_4^2 + b_4^2}}{\sqrt{2}\pi f N_2} 10^8 \text{ gauss,}$$

where Φ' is the maximum flux.

The induced voltage will be in quadrature with the flux producing it so that the phase of the flux will be 90° in advance of E_4. Vectorially, therefore, the flux will be

$$\frac{10^8}{\sqrt{2}\pi f N_2}(-b_4 + ja_4).$$

The energy stored in the flux will be given by

$$P_{j4} = \frac{N_1}{N_2}(a_2 b_4 - a_4 b_2) \text{ reactive volt-amperes.}$$

Similarly the power expended in iron losses to produce this flux density will be

$$P_4 = \frac{N_1}{N_2}(a_2 a_4 + b_2 b_4) \text{ watts.}$$

These two quantities are simply the product of the primary

current and secondary voltage reduced to the primary by the ratio of the turns.

The difference between P_3 and P_4 will be the power loss in the winding or copper loss in the choke, since this difference does not reappear in P_4.

The leakage reactance or flux can also be derived from the difference between P_{j3} and P_{j4} since this is the energy storage associated with both windings. The flux which does not cut the secondary winding does not contribute to E_4. The leakage flux may change under load, when current is flowing in the secondary winding.

The mutual inductance between the primary and secondary winding can be derived from the mutual impedance.

$$\text{That is } Z_M = \frac{E_4}{I}$$

$$= r\frac{+ jb_4}{a_2 + jb_2}.$$

This will be an impure mutual inductance because the secondary voltage is not in quadrature with the primary current, since the flux in the iron is not in phase with the exciting current due to the iron losses.

The mutual impedance will consist of two components:

$$Z_M = r_M + jx_M.$$

$$r_M = r\frac{a_2a_4 + b_2b_4}{a_2^2 + b_2} \text{ ohms.}$$

$$x_M = r\frac{a_2b_4 - a_4b_2}{a_2^2 + b_2^2} \text{ ohms.}$$

The mutual inductance will be $M = \dfrac{x_M}{2\pi f}$ henries.

The impurity will be the effective resistance r_M which can be considered as acting in the circuit, causing the departure from true quadrature of the induced voltage with respect to the primary current.

In dealing with iron circuits, however, the effective mutual inductance derived above would be of little significance, but the method is given for use when air-cored or small loss circuits are measured.

It will be noticed that all the properties of the circuit have been derived without any angles entering into the calculations.

All the vector values can be plotted directly on squared paper from their components. Care must be taken in connexion with the actual signs of the measured values, otherwise entirely erroneous results will be obtained. In practice a quantity is often measured with both signs reversed due to the potential leads being reversed. This gives the measured value an apparent displacement of 180° from its true position, but there is seldom any doubt as to its true polarity if a rough vector diagram is drawn. Both signs can be reversed by a reversal of the potential leads, but never one sign only. The reversal of both signs does not, of course, affect calculated values.

In calculating power and stored energy, the result may give a negative value if one of the quantities has been measured with its signs reversed in this way, but the magnitude will be correct and unless complex circuits are being investigated, the final sign of both power and reactive volt-amperes can be taken as positive so long as the components are treated strictly according to sign in the calculation.

When circuits containing inductance and capacitance or more than one source of power are being measured, great care in the correctness of the recorded signs of components must be taken, because some of the power and reactive volt-ampere measurements will be positive and some negative. All ambiguity is removed at the outset if the polarity of the potential leads is carefully observed. For the correct signs to be obtained, the two potential leads should progress in their same relative positions from point to point round the circuit. That is, the positive lead moves on to a new point, and the negative lead moves to the point previously occupied by the positive lead. In any closed circuit the sum of all inphase or quadrature components should be zero.

When the phase angle of any vector is required, it will be readily found from the ratio of the components.

The phase of the current with respect to the terminal voltage will be given by

$$\psi = \tan^{-1}\frac{b_1}{a_1} - \tan^{-1}\frac{b_2}{a_2}.$$

The phase of the flux with respect to the current will be given by

$$\theta = \tan^{-1}\frac{b_2}{a_2} - \tan^{-1}\frac{a_4}{b_4}.$$

Strict attention must again be paid to the actual measured signs of the components.

There are many cases in which measurement of the voltage or current in co-ordinate form gives a very powerful means of investigation into a.c. circuit phenomena, and for the purpose of illustration some further examples will now be considered.

The torque on the rotor of an induction motor is proportional to the product of the fluxes entering it and the sine of the angle between them. The eddy current motor is a simple example. If a search coil is wound round each half of the eddy current motor poles shown in Fig. 109, a voltage will be induced in each of the search coils when the motor is excited. The torque on the motor disc will be proportional to the product of the two fluxes in the poles and the sine of the angle between them. This is because the flux induces an e.m.f. in the disc which causes a current to flow, and this current is acted upon by the flux in the other pole and vice versa. Since the voltage induced in the disc is in quadrature with the flux inducing it, it is only the quadrature component of the flux which can act upon the current in the disc. The torque is proportional, therefore, to the quadrature product of the fluxes or induced voltages. Thus if E_1 and E_2 are the voltages measured in the two coils, the torque in the disc will be proportional to

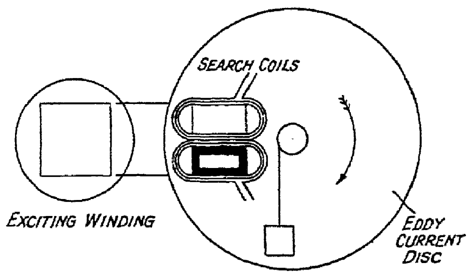


FIG. 109.—Eddy current motor.

$$a_1b_2 - a_2b_1$$

when $\qquad E_1 = a_1 + jb_1$

and $\qquad E_2 = a_2 + jb_2.$

This forms the basis of a simple practical experiment on the theory of the induction motor.

The attenuation of any complex network or circuit can be readily determined from the measurement of the input and output current and voltage.

If the input voltage and current are E_1 and I_2 and the output voltage and current are E_3 and I_4

when $E_1 = a_1 + jb_1$, $I_2 = a_2 + jb_2$, etc.

The input power will be

$$P_1 = a_1a_2 + b_1b_2 \text{ watts.}$$

The output power will be

$$P_3 = a_3a_4 + b_3b_4 \text{ watts.}$$

The ratio of the powers will be

$$\frac{P_1}{P_2} = \frac{a_1a_2 + b_1b_2}{a_3a_4 + b_3b_4}.$$

The attenuation will be

$$10 \operatorname{Log}_{10} \frac{a_1a_2 + b_1b_2}{a_3a_4 + b_3b_4} \text{ decibels.}$$

If the input and output impedances are both equal to Z, the ratio of the input and output powers will be

$$\frac{E_1I_2}{E_3I_4} = \frac{E_1{}^2Z}{E_3{}^2Z}.$$

The attenuation will then be

$$10 \operatorname{Log}_{10} \left(\frac{a_1{}^2 + b_1{}^2}{a_3{}^2 + b_3{}^2}\right) \text{ decibels,}$$

irrespective of the nature of the impedance.

The power dissipated in the network is $P_1 - P_3$

$$= (a_1a_2 + b_1b_2) - (a_3a_4 + b_3b_4) \text{ watts.}$$

The phase-shifting effect of the network on the current is given by

$$\tan^{-1}\frac{b_2}{a_2} - \tan^{-1}\frac{b_4}{a_4}$$

and on the voltage by

$$\tan^{-1}\frac{b_1}{a_1} - \tan^{-1}\frac{b_3}{a_3}.$$

The mutual admittance between input and output is the ratio of the output current to the input voltage.

This is
$$Y_{M_{1-4}} = \frac{a_4 + jb_4}{a_1 + jb_1}$$
$$= \frac{a_1a_4 + b_1b_4}{a_1^2 + b_1^2} + j\frac{a_1b_4 - a_4b_1}{a_1^2 + b_1^2} \text{ mhos.}$$

Similarly the mutual impedance between the output voltage and the input current will be

$$Z_{M_{2-3}} = \frac{a_3 + jb_3}{a_2 + jb_2}$$
$$= \frac{a_2a_3 + b_2b_3}{a_2^2 + b_2^2} + j\frac{a_2b_3 - a_3b_2}{a_2^2 + b_2^2} \text{ mhos.}$$

The mutual admittance or mutual impedance between input and output is not necessarily the same as that between output and input except in a symmetrical network, such as a simple transmission line, a filter, or an even ratio transformer.

The general equations [1-3] governing the relationship between input and output currents and voltages can be easily determined in any network from the measurements on no-load and short-circuit.

The no-load and short-circuit constants measured upon any network, however complex, will give the relationship between the current and voltage at any two parts of the network at which the measurements are made, so long as the network is made up of circuits in which the current is proportional to the voltage. Even when transformers with iron cores are present, the proportionality in many cases is sufficiently close over the working range.

If the relationship is required between the two points which can be called the input and output points respectively, it is necessary to measure the admittance at the input points when the output points are open-circuited and the impedance at the input points when the output points are short-circuited. If the circuit is unsymmetrical, two further quantities are required, these being the ratio of input to the output voltage on open-circuit (*i.e.*, open-circuit voltage attenuation), and the

ratio of the input to output current on short-circuit (*i.e.*, the short-circuit current attenuation).

The four quantities Open-circuit admittance Y_0
Short-circuit impedance Z_K
Voltage attenuation ratio C_1
Current attenuation ratio C_2

are all vector quantities derived from the a.c. potentiometer measurements of voltage and current ; Y_0 will be given by the quotient of $\frac{I_2'}{E_1'}$, when the output is open-circuited,

Z_K by $\frac{E_1''}{I_2''}$ when the output is short-circuited.

C_1 by $\frac{E_1'}{E_3'}$, when the output is open-circuited, and

C_2 by $\frac{I_2''}{I_4''}$ when the output is short-circuited,

where the dashes against the symbols indicate that the particular value of that quantity is used as obtained in the particular condition concerned.

In order to measure I_4'', which is the current passing through the short-circuit, the short-circuit will have to be made by a low resistance of known value. Whether the resistance is sufficiently low to get an accurate result can be tested by altering its value and noting the change in I_4''. A few values will indicate the limiting value to which I_4'' can rise.

When these four constants have been determined in vector form, the relationship between the input and output current and voltage for any load impedance will be given by the equations

$$E_3 = C_2(E_1 - I_2Z_K)$$

and
$$I_4 = C_1(I_2 - E_1Y_0)$$

or conversely
$$E_1 = C_1E_3 + C_2I_4Z_K$$

and
$$I_2 = C_2I_4 + C_1E_3Y_0.$$

If the circuit is symmetrical, it will give the same impedance or admittance value if measured at either the output or the

input points. In this case the above equations are simplified and $C_1 = C_2 = C$

and
$$E_1 = C(E_3 + I_4Z_K)$$
$$I_2 = C(I_4 + E_3Y_0).$$

If the output load admittance Y is known $I_4 = E_3Y$.

$$E_1 = E_3C(1 + YZ_K)$$
and
$$I_2 = E_3C(Y + Y_0).$$

That is, the voltage attenuation will be given by

$$\frac{E_1}{E_3} = C(1 + YZ_K)$$

and the mutual admittance between the input and output, that is, the input current for unit output voltage, will be given by

$$\frac{I_2}{E_3} = C(Y + Y_0).$$

The input impedance to the network will be

$$\frac{E_1}{I_2} = \frac{1 + YZ_K}{Y + Y_0}.$$

The symbolic representation [2] of the double frequency quantities of power and stored energy (or reactive volt-amperes) is such a powerful tool in circuit problems that it is worth while gaining complete familiarity with rules of double-frequency algebra as distinct from the algebra of complex quantities of single frequency. The requirements of a.c. potentiometers are largely covered by the following two products giving the power and reactive volt-amperes corresponding to the measured voltage and current.

If $E = \pm a_1 \pm jb_1$ and $I = \pm a_2 \pm jb_2$ the power

$$P = \pm a_1a_2 \pm b_1b_2 \text{ watts.}$$

The stored energy

$$Pj = \pm a_1b_2 \mp a_2b_1 \text{ reactive volt-amperes}$$
$$= \frac{\pm a_1b_2 \mp a_2b_1}{4\pi f} \text{ joules.}$$

Calculation of Results of Polar Potentiometer Measurements.—When the current and voltage acting in a circuit are measured by means of the polar potentiometer, the results

appear in vector form as a voltage magnitude with a phase angle relationship with respect to some datum vector. The datum vector is arbitrary and can be any convenient one such as the volt drop in a non-inductive resistance representing the current phase in the circuit.

If the voltages in the circuit shown in Fig. 108 are measured, and the voltage on the non-inductive resistance is taken as the datum vector, the angles of the voltages E_1, E_3 and E_4 will be ϕ_2, ϕ_3 and ϕ_4. The magnitude of each of the voltages will be directly determined from a single potentiometer reading.

The current I flowing in the circuit will be $\dfrac{E_2}{r}$.

The total power delivered to the circuit will be $\dfrac{E_1E_2}{r}\cos\phi_2$.

The total reactive volt-amperes will be $\dfrac{E_1E_2}{r}\sin\phi_2$.

The power consumed by the choke will be $\dfrac{E_2E_3}{r}\cos\phi_3$.

The reactive volt-amperes in the choke will be $\dfrac{E_2E_3}{r}\sin\phi_3$.

The power loss in the iron will be $\dfrac{E_2E_4}{r}\cos\phi_4$.

The impedance of the choke will be $\dfrac{E_3}{E_2}r$.

The phase angle of the choke will be ϕ_3,

so that the effective resistance will be $r_3 = \dfrac{E_3}{E_2}r\cos\phi_3$

and the reactance $x_3 = \dfrac{E_3}{E_2}r\sin\phi_3$.

The flux will be given by the relationship

$$\Phi = \frac{10^8 E_4}{\sqrt{2}\pi f N_2},$$

E_4 being the simple numerical value as determined by the potentiometer.

The mutual inductance would be given by

$$M = \frac{E_4}{E_2}\cdot\frac{r}{2\pi f}\cdot\sin\phi_4.$$

The Attenuation of a Circuit or Network.[3]—The attenuation of a circuit will be given by the ratio of the input to the output power. If E_1 and I_2 are the input voltage and current and E_3 and I_4 the output voltage and current. Then

$$\frac{P_1}{P_2} = \frac{E_1 I_2 \cos \phi_1}{E_3 I_4 \cos \phi_3}.$$

The *attenuation* will be

$$10 \operatorname{Log}_{10} \frac{E_1 I_2 \cos \phi_1}{E_3 I_4 \cos \phi_3} \text{ decibels}$$

where ϕ_1 is the phase displacement between E_1 and I_2 and ϕ_3 is the phase displacement between E_3 and I_4.

The calculation of mutual impedance follows in the same way.

$$Z_{M_{2-3}} = \frac{E_3}{I_2}.$$

The phase angle between E_3 and I_2 will be known from the measurement, calling this ϕ_2 the components of $Z_{M_{2-3}}$ will be

$$r_{M_{2-3}} = \frac{E_3}{I_1} \cos \phi_2$$

and
$$x_{M_{2-3}} = \frac{E_3}{I_2} \sin \phi_2.$$

In the calculations of the results obtained from measurements with the polar a.c. potentiometer the trigonometric functions are involved. It will be evident that it is necessary to know the angles with very great precision to obtain accurate values of power where the phase angles are large, and to obtain accurate values of reactive volt-amperes where the phase angles are small. This makes it necessary to triangulate by measuring the three vector voltages which form a closed triangle if precision is required. The triangles so measured can be solved by the ordinary rules of plane trigonometry.

NOTES AND BIBLIOGRAPHY.

[1] La Cour, J. L., and Bragstad: "Theory and Calculation of Electric Currents" (Longmans, Green & Co., 1913).

[2] Steinmetz, C. P.: "Alternating Current Phenomena" (McGraw-Hill Book Company).

[3] Shea, T. E.: "Transmission Networks and Wave Filters" (Chapman & Hall, 1929).

[4] Gall, D. C., and Sims, L. G. A.: "On the Theory and Measurement of the Magnetic Properties of Iron," *J. Instn. Elect. Engrs.*, 1934, lxxiv, 453–68

CHAPTER XIV.

HISTORICAL NOTE ON THE POTENTIOMETER.

The history of the potentiometer is nearly a century old and begins in 1841 when Poggendorff devised a method of measuring the e.m.f. of a cell in terms of another cell without taking current from the former. Very little published data are available concerning the actual apparatus except the circuit and that a slide wire was used. Poggendorff developed the method further in work published in 1847, but did not use a potentiometer in the form as it is understood to-day, for he used a circuit of variable resistance and his determinations involved a knowledge of the internal resistance of one of the batteries. In 1855 Bosscha made the next step forward by eliminating the necessity for determining this resistance. In 1863 du Bois Raymond described the use of a constant resistance slide wire. He had an instrument of his design constructed by Halske in about 1860 which consisted of a circular slide wire rotating against a fixed contact. He had already described a constant resistance voltage divider in 1849. In 1861 Latimer Clark made the first English contribution to the subject and described a number of circuits which were potentiometers in essence in connexion with a Government report on submarine cables. In 1868 Latimer Clark made an important contribution by introducing another cell, which supplied no current, for standardizing purposes but using two galvanometers for balancing. His potentiometer was a long wire wound as a helix on a cylinder which rotated in mercury-filled bearings and had a graduated circumference dividing the wire into 20,000 parts. This was the first introduction of the present-day idea of a standard cell. In the same year Siemens introduced his Universal galvanometer which was a development of the round compensator of Halske-du Bois Raymond.

In 1871 Von Beetz described a null method of measuring the internal resistance of a cell and in the same year Mance described an inverted Poggendorff method of measurement upon the cells of a multiple battery. In 1872 Latimer Clark made his next and important contribution of a real standard cell composed of zinc and mercury in zinc sulphate, and made the first publication of the name "potentiometer" about this time. In his lecture to the Institution of Electrical Engineers in 1873, Latimer Clark claimed to have devised a direct reading potentiometer, but the data on record are not too clear. Further contributions in detail came from Adams in 1874 and Von Beetz in 1878, but in 1880 Pellat brought the circuit arrangement a definite stage nearer to the present form by introducing a selector switch so that only one galvanometer was necessary, the latter being transferred from the standard-cell circuit to the unknown circuit. The circuit was still only partly developed and did not contain means for standardizing at a definite voltage value, but gave a ratio of known to unknown voltages.

In 1885 Lord Rayleigh devised a constant resistance potentiometer circuit by using two resistance boxes in series, and in the same year Fleming used a modified Latimer Clark potentiometer with a Daniel cell for a standard with a single galvanometer and selector switch. In 1886 Fleming devised the measurement of current by the potentiometer and a known resistance and later the use of a volt box. In 1887 he developed the direct reading instrument, practically as used to-day. In 1890 Crompton, at Fleming's suggestion, constructed a long slide-wire instrument with the selector switch for transferring the galvanometer for balancing against the standard cell, and the rheostat to allow for battery variations. In the same year Feussner had constructed the first high-resistance precision potentiometer without a slide wire. The mechanically coupled dials which increase resistance on one side and decrease resistance on the other in the manner of the Rayleigh constant resistance circuit, and often known as the Feussner potentiometer, are really due to Weston, and were patented by him in 1892.

In 1895 Crompton produced the first commercial instrument with a single decade dial and slide wire in the same form

as it is still manufactured. In 1895 R. Franke described a multi-range potentiometer with parallel and series shunts to alter the range, and in the same year Raps applied the Varley slide which the latter had devised for fault localization in 1866. In 1898 Stansfield contributed the deflexional potentiometer with a constant resistance galvanometer circuit.

By this time the instrument was fully established and numerous refinements have been contributed from many sources, but no new principle of measurement has been introduced.

The a.c. potentiometer began in 1891 with the double alternator due to A. Franke. Campbell contributed an a.c. compensator in 1900, Drysdale the phase-shifter and potentiometer in 1908, Larsen introduced his beautifully simple circuit in 1910. Further instruments were described by Erlang in 1913, Pedersen in 1919, Gall in 1923, Geyger in 1924, Campbell in 1928. Many others have also contributed to the subject of a.c. potentiometry.

INDEX.